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MHD mixed convection for viscoelastic fluid past a porous wedge

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ABSTRACT

A magnetic hydrodynamic (MHD) mixed convective heat transfer problem of a second-grade viscoelastic fluid past a wedge with porous suction or injection has been studied. Governing equations include continuity equation, momentum equation and energy equation of the fluid. It has been analyzed by a combination of a series expansion method, the similarity transformation and a second-order accurate finite-difference method. Solutions of wedge flow on the wedge surface have been obtained by a generalized Falkner–Skan flow derivation. Some important parameters have been discussed by this study, which include the Prandtl number (Pr), the elastic number (E), the free convection parameter (G) and the magnetic parameter (M), the porous suction and injection parameter (C) and the wedge shape factor (β). Results indicated that elastic effect (E) in the flow could increase the local heat transfer coefficient and enhance the heat transfer of a wedge. In addition, similar to the results from Newtonian fluid flow and conduction analysis of a wedge, better heat transfer is obtained with a larger G and Pr.

1. Introduction

The analysis of the flow field in a boundary layer adjacent to the wedge is very important in the present problem, and is an essential part in the area of fluid dynamics and heat transfer. Especially, understanding boundary layer flows and heat transfer of non-Newtonian fluids has become important recently [1]. Srivatsava [2] and Rajeswari and Rathna [3] studied non-Newtonian fluid flow near a stagnation point. Mishra and Panda [4] analyzed the behavior of second-grade viscoelastic fluids under the influence of a sidewall injection in an entrance region of a pipe flow. Rajagopal et al. [5] studied a Falkner–Skan flow field of a second-grade viscoelastic fluid. Massoudi and Ramezan [6] studied a wedge flow with suction and injection along the walls of a wedge by the similarity method and finite-difference calculations. Hsu et al. [7] also studied the flow and heat transfer phenomena of an incompressible second-grade viscoelastic fluid past a wedge with suction or injection. An excellent review of boundary layers in non-linear fluids was recently written by Rajagopal [8]. These are related studies to the present investigation about second-grade viscoelastic fluids. The system to be analyzed in the present study is a wedge in a second-grade viscoelastic fluid flow. Due to the coupling nature between the wedge and the fluid, the present analysis is different from previous researches concerning mixed convection about a wedge. Those studies have dealt primarily with a plate having prescribed convective heat transfer coefficient that yield similar or non-similar solutions [9–11]. Recently, there are some related studies [13–16] on wedge flow for micropolar fluid, viscoelastic fluid, non-Darcy mixed convection, and compressible turbulent boundary layer and unsteady mixed convection flow. And also, the related MHD and viscoelastic fluids flow are studied by Aliakbar et al. [17] for the influence of thermal radiation on MHD flow of Maxwellian fluids above stretching sheets. Abel et al. [18–20] studied viscoelastic MHD flow and heat transfer over a stretching sheet with viscous and ohmic dissipations, with variable thermal conductivity, non-uniform heat source and radiation and with non-uniform heat source/sink. Salem [22] studied variable viscosity and thermal conductivity effects on MHD flow and heat transfer in viscoelastic fluid over a stretching sheet. Cortell [23] studied the effects of viscous dissipation and work done by deformation on MHD flow and heat transfer of a viscoelastic fluid over a stretching sheet.

The objective of the present analysis is to study the MHD mixed convection of a wedge cooled or heated by a high or low Prandtl number, second-grade viscoelastic fluid with various parameters, such as Prandtl number (Pr), the elastic number (E), the free convection parameter (G) and the magnetic parameter (M), the porous suction and injection parameter (C) and the wedge shape factor (β). An extension of previous works was then performed to investigate numerical calculation MHD mixed convection for viscoelastic fluid past a wedge with porous suction or injection. A schematic diagram of the wedge is shown in Fig. 1 to illustrate the physical situation and symbols of parameters

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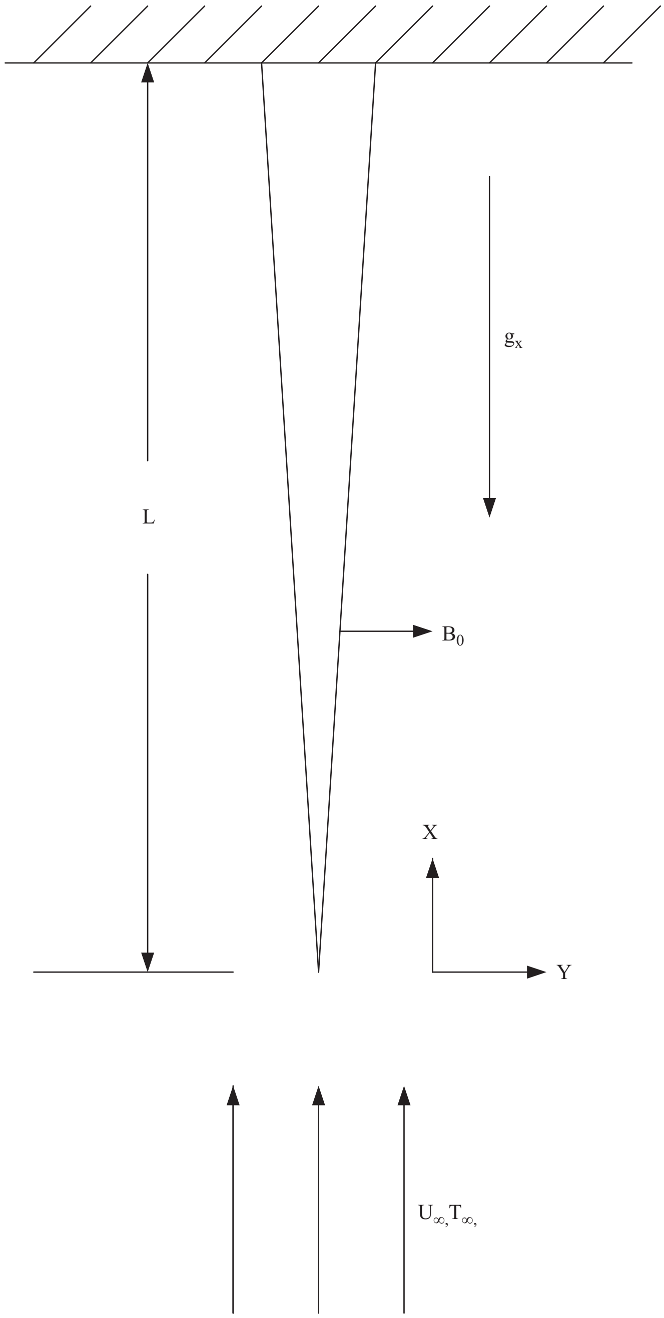

Fig. 1. A schematic diagram for the MHD mixed convection of viscoelastic fluid past a wedge with porous suction or injection.

needed for the analysis. The Rivlin–Ericksen model for grade-two fluids is used in the momentum equations. This work studied the effects of dimensionless parameters, the Prandtl number (Pr), the elastic number (E), the free convection parameter (G) and the magnetic parameter (M). Flow and temperature fields of wedge flow have been analyzed by utilizing the boundary layer concept to obtain a set of coupled momentum equations and energy equations. A similar transformation with wedge-type parameters and a series expansion method are then used to convert the non-linear, coupled partial differential equations to a set of non-linear, decoupled ordinary differential equations. In the present problem, these decoupled equations are then solved iteratively to obtain the dimensionless velocity and dimensionless temperature distribution along the wedge boundary layer by a second-order finite difference method.

2. Theory and analysis

The Rivlin–Ericksen model [24] for a homogeneous, non-Newtonian, second-grade viscoelastic fluid has been used in the present wedge flow. It is important to discuss that the coefficients that characterize the fluid have to satisfy certain restrictions, see Dunn and Rajagopal [25]. In the study, they establish several new results concerning the thermodynamics of these materials. A special application of their results reveals that the work of Joseph [26,27] and Renardy [28] on the instability of the rest state for certain, very special grade n fluids is in fact inapplicable to all those grade n fluids that are consistent with thermodynamics. The model equation is expressed as follows:

$$\mathbf{T} = -P\mathbf{I} + \mu\mathbf{A}_1 + \alpha_1\mathbf{A}_2 + \alpha_2\mathbf{A}_1^2 \tag{1}$$

where P is pressure, $\mathbf{I}$ the unit vector, μ is dynamic viscosity, and α_1 and α_2 are first and second normal stress coefficients that related to the material modulus. The kinematic tensors $\mathbf{A}_1$ and $\mathbf{A}_2$ are defined as

$$\mathbf{A}_1 = \text{grad}\mathbf{V} + (\text{grad}\mathbf{V})^{\mathrm{T}} \tag{2}$$

$$\mathbf{A}_2 = \frac{\mathrm{d}}{\mathrm{d}t}\mathbf{A}_1 + \mathbf{A}_1(\text{grad}\mathbf{V}) + (\text{grad}\mathbf{V})^{\mathrm{T}}\mathbf{A}_1 \tag{3}$$

The model equation (1) is called a second order fluid model if it is not required to be compatible with thermodynamics [30,34]. The sign of the coefficient α_1 has been a subject of much controversy. Dunn and Fosdick [29] demonstrated that a second grade fluid exhibits acceptable stability characteristics. Later, Fosdick and Rajagopal [31] showed that the fluid exhibited anomalous behavior not to be expected of any fluid of rheological interest if $\alpha_1 < 0$ and $\alpha_1 + \alpha_2 \neq 0$. It is very important that solutions to steady flow problems can be found when $\alpha_1 < 0$. However, all these flows are not stable [25]. Several such solutions correspond to the case $\alpha_1 < 0$ presented in the recent book of Truesdell and Rajagopal [32]. As a result, in any event, the results established for the case $\alpha_1 > 0$ have more value than the solution for $\alpha_1 < 0$. In this study, we shall assume that the model under consideration meets $\mu \geq 0$, $\alpha_1 \geq 0$, $\alpha_1 + \alpha_1 = 0$ and is compatible with the present literature, where $\mathbf{V}$ is velocity and d/dt is the material time derivative. As done by Rajagopal [33], the present researchers substituted Eq. (1) into momentum equations

$$\rho\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{V} = \text{div}\mathbf{T} + \rho\mathbf{b} \tag{4}$$

where ρ is the density of the fluid, and assumed that the fluid is incompressible and the flow is in isochoric motion to obtain

$$\text{Div}\mathbf{V} = 0 \tag{5}$$

For the steady, two-dimensional laminar flow under conservative body force $\mathbf{b}$, the following were defined:

$$P^* = P - \left(2\alpha_1 + \frac{\alpha_2}{2}\right)\left(\frac{\partial u}{\partial y}\right)^2 + \rho\Phi \tag{6}$$

$$\mathbf{b} = \nabla\Phi \tag{7}$$

where P, P^* is pressure and Φ is potential function. From Bernoulli's principle and the substitution of the edge velocity u_e, the following equation is obtained:

$$u_e\frac{\partial u_e}{\partial \chi} = -\frac{1}{\rho}\frac{\partial P^*}{\partial \chi} \tag{8}$$

In Eq. (10), use the Oberbeck–Boussinesq approximation. This has not been rigorously justified even in the case of the classical Navier–Stokes fluid and definitely not in the case of the second grade fluid. In 1996 Rajagopal, Ruzicka and Srinivasa provided a justification for the approximation within the full thermodynamical theory for Navier–Stokes fluids [34]. We follow