International Journal of Non-Linear Mechanics 46 (2011) 79–95

Contents lists available at ScienceDirect

International Journal of Non-Linear Mechanics

journal homepage: www.elsevier.com/locate/nlm

Large-deflection and post-buckling behavior of slender beam-columns with non-linear end-restraints

Carlos Vega-Posada [a], Mauricio Areiza-Hurtado [b], J. Dario Aristizabal-Ochoa [c,*]

[a] *Department of Civil and Environmental Engineering, Northwestern University, Evanston, IL, USA*
[b] *ACCIONA Infraestructuras, R&D Department, Composite Group, Madrid, Spain*
[c] *School of Mines, National University of Colombia at Medellín, Colombia*

ARTICLE INFO

Article history:
Received 27 February 2008
Received in revised form
16 July 2010
Accepted 23 July 2010

Keywords:
Bars
Beams
Beam-columns
Buckling
Columns
Composites
Cracking
Large deflections
P–Δ analysis
Post-buckling
Non-linear analysis
Second-order analysis
Semi-rigid connections
Stability

ABSTRACT

The large-deflection analysis and post-buckling behavior of laterally braced or unbraced slender beam-columns of symmetrical cross section subjected to end loads (forces and moments) with both ends partially restrained against rotation, including the effects of out-of-plumbness, are developed in a classical manner. The classical theory of the "Elastica" and the corresponding elliptical functions utilized herein are those presented previously by Aristizabal-Ochoa [1]. The proposed method can be used in the large-deflection analysis and post-buckling behavior of elastic slender beam-columns with rigid, semi-rigid, and simple flexural connections at both ends including linear and non-linear inelastic connections like those that suffer from flexural degradation (such as flexural cracking and elasto-plastic connections) or flexural stiffening. Only bending strains are considered in the proposed analysis. Results from the proposed method are theoretically exact from small to very large curvatures and transverse and longitudinal displacements for laterally braced or unbraced slender beam-columns under bending caused by end loads. The large-deflection analysis and post-buckling behavior of slender beam-columns with both supports partially restrained against rotation and with sway inhibited or uninhibited are complex problems requiring the simultaneous solution of two coupled non-linear equations with elliptical integrals whose unknowns are the limits of the integrals. The validity of the proposed method and equations are verified against solutions available in the technical literature. Three comprehensive examples are included that show the effects of linear and non-linear connections at both ends on the large-deflection analysis and post-buckling behavior of slender beam-columns.

1. Introduction

The non-linear large-deflection analysis and post-buckling behavior of bars and beam-columns are of great importance in physics, structural engineering and engineering mechanics particularly when they are slender and made of high-strength elastic materials like FRP or composites. This problem has been investigated using the exact expression for the curvature in the differential equation of the deflection curve (i.e., the "Elastica" approach, Timoshenko and Gere [2], pp. 76–82) or using the approximate second-order analysis such as the Finite Element Method (FEM) with large deflections and with or without large strains ([3–6], among others).

The exact post-buckling elastic behavior of a perfect beam-column under a gravity load is known; however, its response under other load combinations and the relationships between the applied axial load and the transverse deflections, particularly when the member suffers from imperfections (such as lack of straightness, out-of-plumbness or eccentricity in the applied axial loads) become even more complicated [7]. Khamlichi et al. [8] presented the different formulations to the solution of the large-deflection problem of a perfectly hinged–hinged elastic bar without sidesway under end axial loads, and discussed the effects of the axial strains and shear deformations using the asymptotic expansion technique on the post-buckling behavior.

Aristizabal-Ochoa [9] developed an approximate algorithm based on the classical Timoshenko stability functions for the large-deflection small-strain analysis of beam-columns with semi-rigid connections including the effects of out-of-plumbness, axial strains and lateral bracing. Recently, Aristizabal-Ochoa [1] presented an analytical method for the non-linear large-deflection stability of a slender beam-column of symmetrical cross section with semi-rigid connections (linear and non-linear) and sidesway uninhibited under end

* Corresponding author.
E-mail address: jdaristi@unal.edu.co (J. Dario Aristizabal-Ochoa).

0020-7462/$ - see front matter
doi:10.1016/j.ijnonlinmec.2010.07.006

Nomenclature

The following symbols are used in this publication:

A cross-sectional area of beam-column member
E Young's modulus of the material
h initial length of the beam-column AB
I principal moment of inertia of the beam-column cross-section about its axis of bending
M bending moment at s along the beam-column $= -P_a(y'_a - y') - M_a$
M_a total bending moment applied at A
M_{con} moment applied at the connection
M_{ua} and M_{ub} ultimate moment applied at A and B, respectively
n_a and n_b shape parameter of non-linear connections at A and B, respectively
P_a applied load at A
$P_e = P_{cr}$ Euler load
R_a and R_b stiffness indices of the flexural connections at ends A and B, respectively
x'_a and y'_a deflections of end A along the x'- and y'-axis, respectively (Fig. 1a)
κ_a and κ_b stiffness of the flexural connection at ends A and B, respectively
$\lambda + \theta_o$ angle between the x- and x'-axis
ξ current angle between the tangent to the elastic curve at s and the x'-axis
ξ_a and ξ_b end slopes of member AB measured with respect to the x'-axis (Fig. 1b)
φ_{con} relative rotation at the connection
φ_{cona} and φ_{conb} relative rotation at ends A and B, respectively
φ_{oa} and φ_{ob} plastic rotation of reference at ends A and B, respectively
ρ_a and ρ_b fixity factors of the linear flexural connections at ends A and B, respectively
σ bending stress at s along the beam $= Mz/I$
θ_o initial out-of-plumb angle with respect to the beam-column axis (Fig. 1a)

loads including the effects of axial load eccentricities and out-of-plumbness using the classical theory of the "Elastica". Phungpaingan and Chucheepsakul [10] investigated the post-buckling behavior of a simply supported elastic column with various end elastic rotational restraints subjected to compressive end axial load. More recently, Aristizabal-Ochoa [11] developed a complete algorithm based on the "modified" Timoshenko stability functions for the large-deflection analysis and post-buckling behavior of beam-columns with semi-rigid connections subjected to conservative and non-conservative end loads including the effects of out-of-plumbness, shear deformations, the induced transverse component of the applied axial force, axial strains and lateral bracing.

The main objective of this publication is to present the large-deflection analysis and post-buckling behavior of slender beam-columns of symmetrical cross sections with both supports partially restrained against rotation and sidesway totally inhibited or uninhibited subjected to end loads (forces and moments) including the effects of out-of-plumbness. The classical theory of the "Elastica" and the corresponding elliptical functions are utilized as proposed by Aristizabal-Ochoa [1]. The main limitation of the "Elastica" is that only flexural strains are considered. The proposed method can be used in the large-deflection elastic analysis of slender beam-columns with rigid, semi-rigid (including non-linear inelastic connections that suffer from flexural degradation or flexural stiffening including elasto-plastic connections), and simple connections at both ends. The large-deflection stability analysis of slender beam-columns with both supports partially restrained by rotational springs of different stiffness and with sidesway totally inhibited or uninhibited becomes a complex problem requiring the simultaneous solution of two coupled non-linear equations with elliptical integrals whose unknowns are the limits of the integrals. The validity and effectiveness of the proposed method and equations are verified against two different methods proposed by Aristizabal-Ochoa [9,11] and with analytical results reported recently by Gürel and Kisa [12] and Phungpaingan and Chucheepsakul [10]. Three examples are included that show the effects of linear and non-linear rotational connections at both ends on the large stability analyses and post-buckling behavior of slender beam-columns.

2. Structural model

Assumptions: Consider the 2-D prismatic beam-column that connects points A and B shown in Fig. 1a. The element is made up of the beam-column itself AB, and the flexural connections AA' and BB' at the top and bottom ends, respectively. The flexural connections AA'

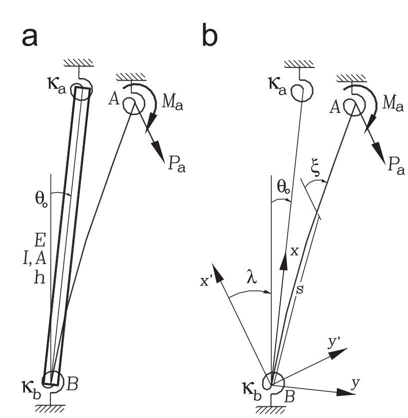

Fig. 1. Beam-column with semi-rigid connections at both ends and with initial out-of-plumbness θ_o: (a) structural model and applied force P_a and moment M_a and (b) coordinate systems and notation.