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Stability and non-linear second-order elastic analyses of beam and framed structures with semi-rigid connections using the cross method

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ABSTRACT

The main objective of this publication is to present an extended version of the Moment Distribution Method (MDM) for the stability and non-linear second-order analysis of indeterminate beams and framed structures made of beam-columns of symmetrical cross-section including the combined effects of shear and bending deformations, axial loads, and semi-rigid connections. The proposed method along each member has the following advantages: (1) it can be utilized in the first- and second-order analyses (including buckling analysis) of indeterminate beams and framed structures made of beam-columns with rigid, semi-rigid, and simple end connections; (2) the effects of semi-rigid connections are condensed into the bending stiffness and fixed-end moments without introducing additional degrees of freedom and equations of equilibrium; and (3) it is accurate, powerful, practical, versatile, and an excellent teaching tool. Analytical studies indicate that shear deformations, semi-rigid connections, and axial loads increase the lateral deflections and affect the internal moments and reactions of continuous beams and framed structures. These effects must be taken into account particularly in slender structures and when they are made of beam or columns with relatively low effective shear areas (like laced columns, columns with batten plates or with perforated cover plates, and columns with open webs) or with low shear stiffness (like short columns made of laminated composites with low shear modulus G when compared to their elastic modulus E) making the shear stiffness GA_s of the same order of magnitude as EI/L^2. These effects become even more significant when the external supports are not perfectly clamped. Three comprehensive examples are included that show the effectiveness of the proposed method.

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1. Introduction

The slope-deflection method and the Hardy Cross method represent the starting points in the evolution and development of the matrix stiffness method as it is known today [1]. The slope-deflection method was presented in 1915 by Wilson and Maney [2] in a *Bulletin* from the University of Illinois at Urbana-Champaign as a general method to be used in the analysis of beam structures with rigid joints subjected to transverse loads. Later, Hardy Cross [3,4] also from the University of Illinois at Urbana-Champaign proposed the first numerical method used in the structural analysis of indeterminate rigid frames that he called "the moment distribution method". The great merit of the Cross method is that it made possible the efficient and safe design of many buildings and rigid jointed frames over half a century [5,6].

In the Cross method it is assumed that the rigid joints of frame members are initially fixed against rotation. The fixed-end moments produced by external loads are computed first as well as the distribution and carry-over factors of each member. These fixed-end moments are unbalanced at the joints of the original non-restrained structure. In order to have rotational equilibrium at each joint, the moment is distributed proportionally to the corresponding member stiffness. These distributed moments are associated with the so-called "carry-over" moments at the opposite ends of structural members. They are considered to be new incremental unbalanced moments and the procedure repeats until the unbalanced moments become negligible. The true moments at the ends of all members are the sum of all distributed moment increments. Initially, the Cross method assumes joint rotations only. However, for frames with joint translations a more general scheme is used, which requires the application of the method successively by setting up a system of equations with the joint translations or sways as unknowns.

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Nomenclature

A_s effective shear area of the beam–column cross-section
E Young's modulus of the material
G shear modulus of the material
L length of the beam–column JK
I principal moment of inertia of the beam–column about its axis of bending
M_j and M_k bending moments (clockwise +) at ends J and K, respectively
P applied axial load at J and K (+compression, −tension)
P_{cr} critical axial load
$P_e = \pi^2 EI/L^2$ euler load
R_j and R_k stiffness indices of the flexural connection at J and K, respectively
$u(x)$ lateral deflection of the beam–column center line
$\beta = 1/(1+P/(GA_s))$ shear reduction factor
Δ sway of end K with respect to end J
κ_j and κ_k flexural stiffness of the end connections at J and K, respectively
λ $EI/(GA_s)$
ρ_j and ρ_k fixity factors at J and K of beam–column JK, respectively
$\psi(x)$ rotation of the cross-section due to bending alone as shown by Fig. 1c
$\psi_{j'}$ and $\psi_{k'}$ bending rotations of cross-sections at ends J′ and K′ with respect to cord J′K′, respectively
$\phi = \sqrt{|P/(\beta EI/L^2)|}$ stability function in the plane of bending
θ_j and θ_k rotations of ends J and K due to bending with respect to the vertical axis, respectively
$\Gamma = 12(EI/L^2)/GA_s$ bending to shear coefficient

On the other hand, advances in composite materials of high resilience capacities and low ratio of shear to bending stiffness of section as well as the need for lighter and stronger beams and columns have created a great interest in the shear effects and second-order analysis of framed structures. The slope-deflection equations for Timoshenko beams including the effects of shear deformations and transverse loads were developed by Bryant and Baile [7]. Previously, Lin et al. [8] had developed the slope-deflection equations for laced and battened beam-columns including the effects of shear deformations, axial loads, and end rigid stay plates. The slope-deflection equations for Timoshenko beams including the effects of shear and bending deformations, second-order $P-\Delta$ effects, and semi-rigid end connections have been presented recently by Aristizabal-Ochoa [9,10] using and the classical stability functions.

The main objective of this publication is to present a new version of the classic moment distribution method for the first- and second-order analyses (including stability) of framed structures made of beam-columns of symmetrical cross-section including the effects of: (1) bending and shear deformations; (2) the shear component of the applied axial forces (Haringx's Model); and (3) semi-rigid connections at the ends of each member. Three comprehensive examples are included that show the effectiveness of the proposed method and corresponding equations.

2. Structural model

2.1. Assumptions

Consider a 2-D prismatic beam–column that connects points J and K as shown in Fig. 1a. The element is made up of the beam–column itself J′K′, and the flexural connections JJ′ and KK′ with bending stiffness κ_j and κ_k at ends J and K, respectively. It is assumed that the beam–column J′K′ of span L bends about the principal axis of its cross-section with a moment of inertia I, effective shear area A_s, and: (1) is made of a homogeneous linear elastic material with the Young and shear moduli E and G, respectively; (2) its centroidal axis is a straight line; and (3) is loaded at both ends with P (axial load) along its centroidal axis.

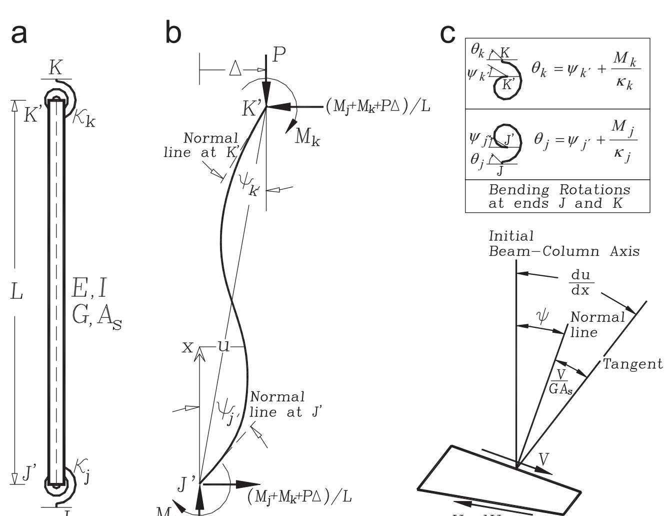

Fig. 1. Beam–column under end moments with semi-rigid connections: (a) structural model; (b) degrees of freedom, forces and moments in the plane of bending; and (c) rotations at a cross-section and at ends A and B.

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