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Two-phase non-linear model for blood flow in asymmetric and axisymmetric stenosed arteries

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ABSTRACT

The pulsatile flow of a two-phase model for blood flow through axisymmetric and asymmetric stenosed narrow arteries is analyzed, treating blood as a two-phase model with the suspension of all the erythrocytes in the core region as the Herschel–Bulkley material and plasma in the peripheral layer as the Newtonian fluid. The perturbation method is applied to solve the resulting non-linear implicit system of partial differential equations. The expressions for various flow quantities are obtained. It is found that the pressure drop, plug core radius, wall shear stress increase as the yield stress or stenosis height increases. It is noted that the velocity increases, longitudinal impedance decreases as the amplitude increases. For asymmetric stenosis, the wall shear stress increases non-linearly with the increase of the axial distance. The estimates of the increase in longitudinal impedance to flow of the two-phase Herschel–Bulkley material are significantly lower than those of the single-phase Herschel–Bulkley material. The results show the advantages of two-phase flow over single-phase flow in small diameter arteries with stenosis.

1. Introduction

Blood is an important biofluid which is the suspension of red blood cells (RBCs), white blood cells (WBCs), platelets and a variety of lipoproteins in aqueous plasma. Plasma is an aqueous solution of various proteins, clotting factors and various ions [1]. Red blood cells are very numerous than white blood cells and are morphologically very simple. They contain hemoglobin which transports oxygen around the body [2]. Platelets are very small, but extremely important in relation to blood coagulation [3].

The clot formation occurs due to the causes like the endothelial injury, endothelial dysfunction, flow stagnation, etc. [4]. Clots are formed at the end of a series of interacting biochemical processes: platelet adhesion, activation and aggregation, coagulation (extrinsic and intrinsic), polymerization of fibrin monomers formed from fibrinogen, and cross linking of the fibrin polymer strands to form a fibrin network [5,6]. Fogelson [7] analyzed a continuum model for platelet aggregation and investigated its mechanical properties. Fogelson and Guy [8] further extended these continuum models to analyze the platelet–wall interactions of platelet thrombosis, using numerical solution.

Lawson et al. [9] analyzed the complex-dependent inhibition of factor VIIa by antithrombin III and heparin. Lawson et al. [10] developed an experimental model for the tissue factor pathway to thrombin. Attaullakhanov et al. [11] experimentally studied the spatio-temporal dynamics of blood coagulation and pattern formation. Mann et al. [12,13] developed models for blood coagulation and the dynamics of thrombin formation. Panteleev et al. [14] formulated mathematical models for the study of blood coagulation and platelet adhesion in their review and provided some clinical applications of the mathematical models.

As the significant devotion to the study of shear-thinning viscoelastic nature of blood, Thurston [15] investigated an extended Maxwell model for the one-dimensional flow of blood. Anand and Rajagopal [16] developed a shear-thinning viscoelastic fluid model for blood flow within a thermodynamic framework that takes cognizance of the fact that viscoelastic fluids can remain stress free in several configurations. Anand et al. [5] analyzed a viscoelastic model within the thermodynamic frame of reference for analyzing the mechanics of a coarse ligated plasma dot.

Blood flow through stenosed arteries has been investigated widely [17–19], because, fluid dynamics plays an important role in the progression of arteriosclerosis and infarcts. The development of arteriosclerosis in blood vessels is quite common, which may be attributed to the accumulation of lipids in the arterial wall or pathological changes in the tissue structure [20]. When an obstruction is developed in an artery, one of the most serious consequences is the increased resistance and the associated reduction of the blood flow to the particular vascular bed supplied by the artery [21]. Thus, the presence of a stenosis can lead to the serious circulatory disorder.

Several theoretical and experimental attempts have been made to study the blood flow characteristics due to the presence

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of a stenosis in the arterial lumen of a blood vessel [22–27]. Although most of the previous investigations [28–31] analyzed blood flow in larger arteries, the occlusion of small arteries is also important. Lacunar infarcts are small and deep cerebral infarcts resulting from the occlusion of penetrating cerebral arteries, which have a diameter of about 100 μm [32]. Even such a small infarct can causes serious damage to the brain [33–37]. Hence, the study of the blood flow through a narrow stenosed artery is very important.

Since, the blood flow through narrow arteries is highly pulsatile, several attempts were made to analyze the pulsatile flow of blood treating blood as a Newtonian fluid [24,25,29,37]. Newtonian approximation provides good results for blood flow in large arteries. But, blood, being the suspension of red cells in plasma, exhibits non-Newtonian behavior at low shear rates ($\dot{\gamma} < 10/\mathrm{s}$) in small diameter arteries (0.02–0.1 mm). In diseased state, the actual blood flow is distinctly pulsatile [38–41]. Several attempts have been made to study the non-Newtonian behavior and pulsatile flow of blood through stenosed tubes [19,22,24,27,42]. Sankar and Lee [43] studied the pulsatile flow of the H–B (Herschel–Bulkley) material for blood flow through asymmetric and axially symmetric stenosed blood vessels.

Misra and Pandy [44] and Chakravarthy et al. [45] have mentioned that for blood flowing through narrow blood vessels, there is a peripheral layer of plasma (a Newtonian fluid) and a core region of suspension of all the erythrocytes as a non-Newtonian fluid. Thus, for a realistic description of blood flow, perhaps, it is more appropriate to treat blood as a two-phase model consisting of a core region (central layer) containing all the erythrocytes as a non-Newtonian fluid and a peripheral layer of plasma as a Newtonian fluid. Several researchers have studied the two-phase models for blood flow through stenosed arteries treating the fluid in the core region as a non-Newtonian fluid and the fluid in the peripheral layer as a Newtonian fluid [30,31,46,47]. Several researchers [45,48,49] have analyzed two-phase non-linear mathematical models for blood flow through axially symmetric stenosed arteries. In this paper, we have studied a two-phase model for pulsatile flow of blood through asymmetric and axisymmetric stenosed arteries, treating the suspension of all the erythrocytes in the core region as the Herschel–Bulkley (H–B) material and plasma in the peripheral layer as a Newtonian fluid.

The layout of the paper is as follows: Section 2 formulates the problem mathematically and then non-dimensionalizes the governing equations and boundary conditions. In Section 3, the resulting non-linear coupled implicit system of differential equations is solved using the perturbation method. The expressions for the velocity, flow rate, wall shear stress, plug core radius and longitudinal impedance to flow have been obtained. Section 4 analyses the variations of these flow quantities with stenosis height, stenosis shape, yield stress, amplitude, power law index and pulsatile Reynolds number ratio through appropriate graphs. The estimates of the increase in the longitudinal impedance to flow for the two-phase model and single-phase model over the uniform diameter tube for different values of the stenosis height and stenosis shape are calculated. Some of the main results are summarized in Section 5.

2. Mathematical formulation

Consider an axially symmetric, laminar, pulsatile and fully developed flow of blood (assumed to be incompressible) in the $\bar{z}$ direction through a circular artery with an axially asymmetric mild stenosis. The walls of the artery are assumed to be rigid and blood is represented by a two-phase model with the core region of suspension of all erythrocytes as the Herschel–Bulkley material and the peripheral layer of plasma as a Newtonian fluid. Fig. 1 shows the geometry of the stenosis artery. Cylindrical polar coordinate system $(\bar{r},\bar{\phi},\bar{z})$ is used to analyze the flow.

It can be shown that the radial velocity is negligibly small and can be neglected for a low Reynolds number flow in a tube with mild stenosis. In this case, the basic momentum equations governing the flow are

$$\bar{\rho}_H(\partial\bar{u}_H/\partial\bar{t}) = -(\partial\bar{p}/\partial\bar{z}) - (1/\bar{r})(\partial(\bar{r}\bar{\tau}_H)/\partial\bar{r}) \quad \text{in} \quad 0 \le \bar{r} \le \bar{R}_1(\bar{z}) \tag{1}$$

$$\bar{\rho}_N(\partial\bar{u}_N/\partial\bar{t}) = -(\partial\bar{p}/\partial\bar{z}) - (1/\bar{r})(\partial(\bar{r}\bar{\tau}_N)/\partial\bar{r}) \quad \text{in} \quad \bar{R}_1(\bar{z}) \le \bar{r} \le \bar{R}(\bar{z}) \tag{2}$$

$$0 = -(\partial\bar{p}/\partial\bar{r}) \tag{3}$$

where the shear stress $\bar{\tau} = |\bar{\tau}_{\bar{r}\bar{z}}| = -\bar{\tau}_{\bar{r}\bar{z}}$ (since $\bar{\tau} = \bar{\tau}_H$ or $\bar{\tau} = \bar{\tau}_N$). The relations between the shear stress and the strain rate of the fluids in motion in the core region (for the Herschel–Bulkley material) and in the peripheral region (for a Newtonian fluid) are given by

$$\bar{\tau}_H = \sqrt[n]{\bar{\mu}_H(\partial\bar{u}_H/\partial\bar{r})} + \bar{\tau}_y \quad \text{if} \quad \bar{\tau}_H \ge \bar{\tau}_y \quad \text{and} \quad \bar{R}_P \le \bar{r} \le \bar{R}_1(\bar{z}) \tag{4}$$

$$(\partial\bar{u}_H/\partial\bar{r}) = 0 \quad \text{if} \quad \bar{\tau}_H \le \bar{\tau}_y \quad \text{and} \quad 0 \le \bar{r} \le \bar{R}_P \tag{5}$$

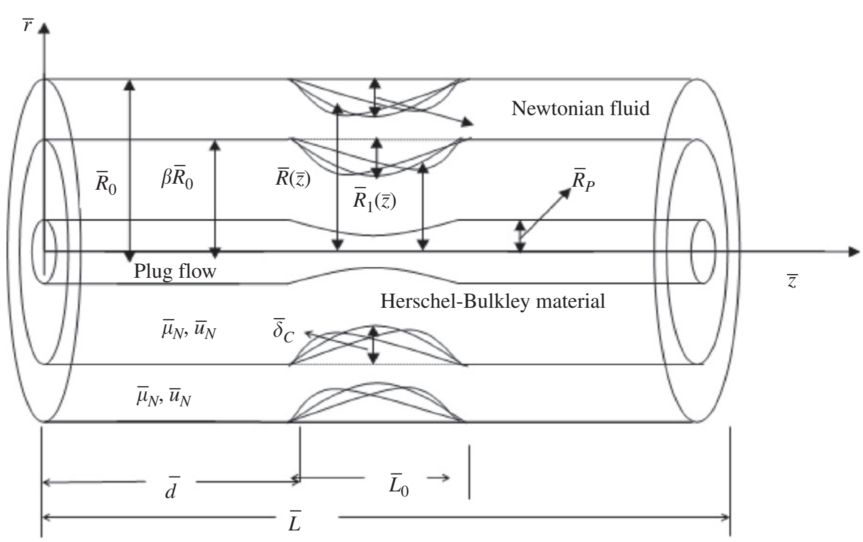

Fig. 1. Geometry of the stenosed artery.