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Reliability evaluation of a port oil transportation system in variable operation conditions

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Abstract

The semi-Markov model of the system operation processes is proposed and its selected parameters are determined. The series 'm out of k_n' multi-state system is considered and its reliability and risk characteristics are found. Next, the joint model of the system operation process and the system multi-state reliability and risk is constructed. Moreover, reliability and risk evaluation of the multi-state series 'm out of k_n' system in its operation process is applied to the port oil transportation system.

Keywords: Reliability function; Oil transportation; Pipelines

1. Introduction

Many technical systems during their operation fulfil variable in time task repertory that results in varying in time their operation states. Sometimes, the designated task repertory in a fixed operation process is performed by some selected and not all subsystems of these systems. The task repertory also has the influence on the component reliability functions of these systems. Thus, we meet here the systems with variable in time their reliability structures and variable their components reliability functions. One of the useful approaches in such systems, reliability analysis and evaluation is applying the semi-Markov model of these systems operation states changing. In this model, the variability of system components reliability characteristics is pointed by introducing the components' conditional reliability functions determined by the system operation states. This approach will be illustrated by the reliability analysis of the series 'm out of k_n' port oil transportation system.

2. System operation process

We assume that the system during its operation process is performing a repertory of tasks. Namely, the system at each moment t, $t \in \langle 0,\theta \rangle$, where θ is its operation time, is performing at most w tasks. We denote the system operation process by

$$Z(t) = [Z_1(t), Z_2(t), \ldots, Z_w(t)],$$

where $Z_j(t) = 1$ if the system is executing the jth task and $Z_j(t) = 0$ if the system is not executing the jth task for $j = 1,2,\ldots,w$. Thus, $Z(t)$ is the process with continuous time t, $t \in \langle 0,\theta \rangle$, and discrete states from the set of states $\{0, 1\}^w$. We number the operation states of the process $Z(t)$ assuming that it have v different states from the set

$$Z = \{z^1, z^2, \ldots, z^v\},$$

and they are of the form

$$z^b = \left[z_1^b, z_2^b, \ldots, z_w^b\right], \quad b = 1,2,\ldots,v,$$

where $z_j^b \in \{0,1\}$, $j = 1,2,\ldots,w$. In practice, a convenient assumption is that $Z(t)$ is a semi-Markov process [1] with its conditional sojourn times θ^{bl} at the operation state z^b when its next operation state is z^l, b, $l = 1,2,\ldots,v$, $b \neq l$. In this case, this process may be described by:

- the vector of probabilities of the initial operation states $[p^b(0)]_{1xv}$,
- the matrix of the probabilities of its transitions between the states $[p^{bl}]_{vxv}$,
- the matrix of the conditional distribution functions $[H^{bl}(t)]_{vxv}$ of the sojourn times θ^{bl}, $b \neq l$.

If the sojourn times θ^{bl}, b, $l = 1,2,\ldots,v$, $b \neq l$, have exponential distributions with the transition rates between the operation states γ^{bl}, i.e. if for b, $l = 1,2,\ldots,v$, $b \neq l$,

$$H^{bl}(t) = P(\theta^{bl} < t) = 1 - \exp[-\gamma^{bl} t], \quad t > 0,$$

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then their mean values are determined by

$$E[\theta^{bl}] = 1/\gamma^{bl}, \quad b, l = 1,2,\ldots,v, \quad b \neq l. \tag{1}$$

The unconditional distribution functions of the process $Z(t)$ sojourn times θ^b at the operation states z^b, $b=1,2,\ldots,v$, are given by

$$H^b(t) = 1 - \sum_{l=1}^{v} p^{bl} \exp[-\gamma^{bl} t], \quad t \geq 0, \quad b = 1,2,\ldots,v, \tag{2}$$

and, considering (1), their mean values are

$$M^b = E[\theta^b] = \sum_{l=1}^{v} p^{bl}/\gamma^{bl}, \quad b = 1,2,\ldots,v, \tag{3}$$

and variations are

$$D[\theta^b] = E[(\theta^b)^2] - E^2[\theta^b] = 2\sum_{l=1}^{v} p^{bl} \frac{1}{(\gamma^{bl})^2} - [M^b]^2, \tag{4}$$

$b = 1,2,\ldots,v.$

Limit values of the transient probabilities $p^b(t)$ at the operation states z^b are given by

$$p^b = \lim_{t\to\infty} p^b(t) = \pi^b M^b \Big/ \sum_{l=1}^{v} \pi^l M^l, \quad b = 1,2,\ldots,v, \tag{5}$$

where M^b are given by (3) and the probabilities π^b of the vector $[\pi^b]_{1xv}$ satisfy the system of equations

$$\begin{cases} [\pi^b] = [\pi^b][p^{bl}] \\ \sum_{l=1}^{v} \pi^l = 1. \end{cases}$$

3. Multi-state series 'm out of k_n' system

In the multi-state reliability analysis [2–10] to define systems with degrading components, we assume that all components and a system under consideration have the reliability state set $\{0,1,\ldots,z\}$, $z \geq 1$, the reliability states are ordered, the state 0 is the worst and the state z is the best and the component and the system reliability states degrade with time t without repair. The above assumptions mean that the states of the system with degrading components may be changed in time only from better to worse ones.

One of basic multi-state reliability structures with components degrading in time are series 'm out of k_n' systems. To define them, we additionally assume that E_{ij}, $i=1,2,\ldots,k_n$, $j=1,2,\ldots,l_i$, k_n, l_1, $l_2,\ldots,l_{k_n}$, $n\in N$, are components of a system, $T_{ij}(u)$, $i=1,2,\ldots,k_n$, $j=1,2,\ldots,l_i$, k_n, l_1, $l_2,\ldots,l_{k_n}$, $n\in N$, are independent random variables representing the lifetimes of components E_{ij} in the state subset $\{u,u+1,\ldots,z\}$, while they were in the state z at the moment $t=0$, $e_{ij}(t)$ are components E_{ij} states at the moment t, $t\in(-\infty,\infty)$, while they were in the state z at the moment $t=0$, $T(u)$ is a random variable representing the lifetime of a system in the reliability state subset $\{u,u+1,\ldots,z\}$ while it was in the reliability state z at the moment $t=0$ and $s(t)$ is the system reliability state at the moment t, $t\in(-\infty,\infty)$, given that it was in the state z at the moment $t=0$.

Definition 1. A vector

$$R_{ij}(t,\cdot) = [R_{ij}(t,0), R_{ij}(t,1), \ldots, R_{ij}(t,z)], \quad t \in (-\infty,\infty),$$

where

$$R_{ij}(t,u) = P(e_{ij}(t) \geq u | e_{ij}(0) = z) = P(T_{ij}(u) > t),$$

for $t\in(-\infty,\infty)$, $u=0,1,\ldots,z$, $i=1,2,\ldots,k_n$, $j=1,2,\ldots,l_i$, is the probability that the component E_{ij} is in the reliability state subset $\{u,u+1,\ldots,z\}$ at the moment t, $t\in(-\infty,\infty)$, while it was in the reliability state z at the moment $t=0$, is called the multi-state reliability function of a component E_{ij}.

Definition 2. A vector

$$\boldsymbol{R}^{(m)}_{k_n l_n}(t,\cdot) = \left[1, \boldsymbol{R}^{(m)}_{k_n l_n}(t,0), \boldsymbol{R}^{(m)}_{k_n l_n}(t,1), \ldots, \boldsymbol{R}^{(m)}_{k_n l_n}(t,z)\right],$$

where

$$\boldsymbol{R}^{(m)}_{k_n l_n}(t,u) = P(s(t) \geq u | s(0) = z) = P(T(u) > t),$$

for $t\in(-\infty,\infty)$, $u=0,1,\ldots,z$, is the probability that the system is in the reliability state subset $\{u, u+1,\ldots,z\}$ at the moment t, $t\in(-\infty,\infty)$, while it was in the reliability state z at the moment $t=0$, is called the multi-state reliability function of a system.

It is clear that from Definitions 1 and 2, for $u=0$, we have $R_{ij}(t,0)=1$ and $\boldsymbol{R}^{(m)}_{k_n l_n}(t,0)=1$.

Definition 3. A multi-state system is called series 'm out of k_n' if its lifetime $T(u)$ in the state subset $\{u, u+1,\ldots,z\}$ is given by

$$T(u) = T_{(k_n-m+1)}(u), \quad u = 1,2,\ldots,z,$$

where $T_{(k_n-m+1)}(u)$ is mth maximal statistics in the random variables set

$$T_i(u) = \min_{1\leq j\leq l_i}\{T_{ij}(u)\}, \quad i = 1,2,\ldots,k_n, \quad u = 1,2,\ldots,z.$$

Definition 4. A multi-state series 'm out of k_n' system is called regular if

$$l_1 = l_2 = \ldots = l_{k_n} = l_n, l_n \in N.$$

Definition 5. A multi-state regular series 'm out of k_n' system is called non-homogeneous if it is composed of a, $1\leq a\leq k_n$, $k_n\in N$, different types of series subsystems and the fraction of the ith type series subsystem is equal to q_i, where $q_i>0$, $\sum_{i=1}^{a} q_i = 1$

Moreover, the ith type series subsystem consists of e_i, $1\leq e_i\leq l_n$, $l_n\in N$, types of components with reliability functions

$$R^{(i,j)}(t,\cdot) = [1, R^{(i,j)}(t,1), \ldots, R^{(i,j)}(t,z)],$$

where

$$R^{(i,j)}(t,u) = 1 - F^{(i,j)}(t,u),$$