International Journal of Non-Linear Mechanics 45 (2010) 902–918

Contents lists available at ScienceDirect

International Journal of Non-Linear Mechanics

journal homepage: www.elsevier.com/locate/nlm

Non-linear dynamics of the sandwich double circular plate system

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ARTICLE INFO

Article history:
Received 18 October 2008
Received in revised form
29 November 2009
Accepted 6 December 2009

Keywords:
Hybrid sandwich non-linear system
Circular plates
Visco-non-linear elastic layer
Multi-frequency
Resonance
Resonant jumps
Mutual interaction
Stability
Trigger

ABSTRACT

Multi-frequency vibrations of a system of two isotropic circular plates interconnected by a visco-elastic layer that has non-linear characteristics are considered. The considered physical system should be of interest to many researches from mechanical and civil engineering. The first asymptotic approximation of the solutions describing stationary and no stationary behavior, in the regions around the two coupled resonances, is the principal result of the authors. A series of the amplitude–frequency and phase–frequency curves of the two frequency like vibration regimes are presented. That curves present the evolution of the first asymptotic approximation of solutions for different non-linear harmonics obtained by changing external excitation frequencies through discrete as well as continuous values. System of the partial differential equations of the transversal oscillations of the sandwich double circular plate system with visco-non-linear elastic layer, excited by external, distributed, along plate surfaces, excitation are derived and approximately solved for various initial conditions and external excitation properties. System of differential equations of the first order with respect to the amplitudes and the corresponding number of the phases in the first asymptotic averaged approximation are derived for different corresponding multi-frequency non-linear vibration regimes. These equations are analytically and numerically considered in the light of the stationary and no stationary resonant regimes, as well as the multi-non-linear free and forced mode mutual interactions, number of the resonant jumps.

1. Introduction

In many engineering systems with non-linearity, high frequency excitations are the sources of multi-frequency resonant regimes appearance high as well as low frequency free and forced modes. That is visible from many experimental research results and also theoretical results (see Nayfeh Ali [39,41,43]). The interaction between amplitudes and phases of the different modes in the non-linear systems with many degrees of the freedom as in the deformable body with infinite numbers frequency vibration free and forced regimes is observed theoretically in Stevanović [58,60] as well as Stevanović and Rašković [59] by use averaging asymptotic methods Krilov–Bogoliyubov–Mitropolyskiy (see Mitropolyskiy [31–38]). This knowledge has great practical importance.

In the monographs by Nayfeh [44,45], a coherent and unified treatment of analytical, computational, and experimental methods and concepts of modal non-linear interactions is presented. This monograph is an obvious extension of Nayfeh's and Balachandran's well-known monograph [42] titled *Applied Non-linear Dynamics* (1995). These methods are used to explore and unfold in a unified manner the fascinating complexities in non-linear dynamical systems. Through the mechanisms discussed in this monograph, energy from high-frequency sources of non-linear phenomena can be transferred to the low-frequency modes of supporting structures and foundations, and the result can be harmful large-amplitude oscillations that decrease their fatigue lives. However, these mechanisms can be exploited to transfer the energy from the main examined system to the designed subsystem and hence decrease considerably the vibrations of the main system and increase its fatigue life.

An experimental and theoretical study of the response of a flexible cantilever beam to an external harmonic excitation near the beam's third natural frequency is presented and in addition, Nayfeh and Malatkar [3] noted that the energy transfer between the third and first modes is very much dependent upon the closeness of the modulation (or Hopf bifurcation) frequency to the first-mode natural frequency. In earlier studies Nayfeh and coworkers [39–45], the modulation frequency was close to the first-mode natural frequency, and therefore large first-mode swaying was observed. Nayfeh developed a reduced-order analytical model by discretizing the integral partial-differential equation of motion. Identifying, evaluating, and controlling dynamical integrity measures in non-linear mechanical oscillators is topic for researchers, see Rega and Lenci [53] and Hedrih [15]. Energy transfer between coupled oscillators can be a measure of the dynamical integrity of

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0020-7462/$ - see front matter © 2009 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ijnonlinmec.2009.12.007

hybrid systems as well as subsystems by Kozmin et al. [24], Hedrih [9,19,20] and Hedrih and Simonović [23].

In the paper Lenci and Rega [26], dimension reduction of homoclinic orbits of buckled beams via the non-linear normal modes technique is presented. The problem of detecting the homoclinic orbits of an initially straight buckled beam is addressed. Two families of boundary conditions are identified and investigated in detail. For the second family, the manifold is no longer planar, and is detected via the non-linear normal modes technique by obtaining approximate expressions which are sufficient to highlight the effects of the non-flatness. A hierarchy of reduced order, single degree of freedom, model is determined. In the series by Lenci and Rega [28,29] and Rega and Lenci [50–52] the problem of detecting the homoclinic orbits applied to the different engineering system dynamics is investigated, and obtained original research results. In Lacarbonara and Rega [25,26], resonant non-linear normal modes in the cases of two-to-one, three-to-one, and one-to-one internal resonances in undamped unforced one-dimensional systems with arbitrary linear, quadratic and cubic non-linearities are investigated for a class of shallow symmetric structural systems. Non-linear orthogonality of the modes and activation of the associated interactions are clearly dual problems.

Alhazza et al. [1] contain research results concerning the non-linear vibrations of parametrically excited cantilever beams subjected to non-linear delayed-feedback control.

In the series of references it is possible to find different approach to obtain solutions of the non-linear dynamics of real systems, as well to discover non-linear phenomena or some properties of the system dynamics as it is: in Shaw and Balachandran [54], a review of non-linear dynamics of mechanical systems in year 2008 is presented; in Singha and Daripa [56], non-linear vibrations of symmetrically laminated composite skew plates by finite element method is investigated; in Shukla and Nath [55], an analytical solution for buckling and post-buckling of angle-ply laminated plates under thermo-mechanical loading is considered; in Rega [50], non-linear vibration of symmetrically laminated composite skew plates by finite element method is considered; in Rega [50], non-linear temporal dynamics of two-mode interactions of magnetized flow is object of investigation; in Singha and Daripa [56], dynamics of finite element structural models with multiple unilateral constraints is considered; in Nayfeh [43], non-linear vibration of heated orthotropic sandwich plates and shallow shells, also in [44] dynamic geometrically non-linear analysis of transversely compressible sandwich plates and in Ebcioglu [2], non-linear theories of sandwich shells are investigated and some new results are presented. List of the valuable research results in connected area of the objects of author research is large, but in this introduction, we pit some subjective choice by us.

Ref. [27] by Lazarov and Thomsen (2009) contain results how high-frequency (HF) excitation combined with strongly non-linear elasticity may influence the effective properties for low-frequency wave propagation. The non-linear normal modes and their bifurcation of a complex two DOF system are investigated systematically and results was published in Ref. [30] by Li et al. (2006). A system of governing equations for non-linear vibrations of heated sandwich shallow shells is derived using Hamilton's principle in combination with Reissner-Hellinger's variational principle and presented in Ref. [46] by Ohnabe (1995). Results of dynamic geometrically nonlinear analysis of transversely compressible sandwich plates are presented in the Ref. [47] by Perel and Palazotto (2003). Investigation of non-linear temporal dynamics of two-mode interactions of magnetized flow is new contributions by Sirwah (2008) presented in Ref. [57]. Dynamics of finite element structural models with multiple unilateral constraints was investigated by Theodosiou and Natsiavas (2009) (see Ref. [61]).

By using averaging and asymptotic methods for obtaining system of ordinary differential equations of amplitudes and phases in first approximations and expressions for energy of the excited modes depending on amplitudes, phases and frequencies of different non-linear modes are obtained by Hedrih [7,9,21,22] and by Hedrih and Simonović [23]. By means of these asymptotic approximations, the energy analysis of mode interaction in the multi-frequency free and forced vibration regimes of non-linear elastic systems (beams, plates, and shells) excited by perturbation of equilibrium state of the system at initial moment defined by initial conditions was made and a series of resonant jumps as well as energy transfer features were identified. Meaning that excitation was by perturbation of equilibrium state of the double plate system at initial moment, defined by initial conditions for displacements and velocities of both plate middle surface points. Also, for the case of an external excitation in the resonant frequency range near one of the natural eigen frequency of the basic linear system, two or more resonant energy jumps at the non-linear modes are present.

Application of the Krilov–Bogoliyubov–Mitropolyskiy asymptotic method as well as energy approach given in monographs by Mitropolyskiy (see Refs. [32,33,36]), by Mitropolyskiy and Mosseenkov (see Refs. [34,35]) for study of the elastic bodies non-linear oscillations and energetic analysis of the elastic bodies oscillatory motions give new results in magister and doctoral theses by Stevanović in 1972 [63] and 1975 [58] (see Refs. [20] as well as [7–23]). The introduction of paper by Hedrih and Simonović [23], presents a review survey of original results of the author and of researchers from Faculty of Mechanical Engineering University of Niš, inspired and/or obtained by the asymptotic method of Krilov–Bogoliyubov–Mitropolyskiy, and a direct influence of professor Rašković [48,49] with his scientific instructions and published Mitropolyskiy's papers and monographs. These results have been published in scientific journals, and were presented on the scientific conferences and in the bachelor degree works [see Stevanović [64]], magister theses [Stevanović [63], Kozić [65], Pavlović [66], Sl.Mitić [67], Pavlov [68], Jovanović [69], Sn. Mitić [70], Filipovski [71], Janevski [72], Simonović [73], Jović [74]…] and doctoral theses [see (Stevanović) [58], Kozić [75], Pavlović [76], Sl.Mitić [77], Knežević [78], Perić [79], Jovanović [80]] supervised by Mitropolskiy (in period from 1972 to 1975) or by Rašković (in period from 1964 to 1974) and by Hedrih in period from 1976 to 2009 year as well.

The interest in the study of coupled plates as new qualitative system dynamics has grown exponentially over the last few years because of the theoretical challenges involved in the investigation of such systems. Recent technological innovations have caused a considerable interest in the study of component and hybrid dynamical processes of coupled rigid and deformable bodies (plates, beams and belts) (see Refs. [13–17,21–23]) denoted as hybrid systems, characterized by the interaction between subsystem dynamics, governed by coupled partial differential equations with boundary and initial conditions.

In the papers Hedrih [16–19], by using examples of hybrid systems of a statically and dynamically coupled discrete subsystem of rigid bodies and continuous subsystem, the method for obtaining frequency equations of small oscillations is presented. Also, series of the theorems of small oscillations frequency equations are defined. By using examples, analogy between frequency equations of some classes of these systems is identified. Special cases of discretization and continoulization of coupled subsystems in the light of these sets of eigen circular frequencies and frequency equations of small oscillations are analyzed.

The study of transversal vibrations of a double like as multi-plates system with elastic, visco-elastic or creep connections is important for both theoretical and pragmatic reason. Many