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ScienceDirect

Acta Materialia 80 (2014) 220–238

Acta MATERIALIA

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Steady spatially-periodic eutectic growth with the effect of triple point in directional solidification

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Received 12 June 2014; accepted 18 June 2014

Abstract

The present paper investigates steady spatially periodic eutectic growth during directional solidification with isotropic surface tension in terms of analytical approach. We consider the case when the Péclet number ϵ is small and the segregation coefficient κ is close to unit, and obtain a family of the global, steady-state solutions with two free parameters: the tilt angle φ and the Péclet number ϵ. The corresponding interfacial patterns of the steady states are spatially periodic, and may be tilted or non-tilted. The results show that near the triple point, there is a boundary layer $O\left(\epsilon^{\frac{1}{2}}\right)$ thick, where the isotropic surface tension plays a significant role, the slope and curvature of interface may be very large and the undercooling temperatures of interface may have a noticeable non-uniformity. Quantitative comparisons between theoretical predictions and recent experimental data are made without making any adjustments to parameters, and show good agreement.

Keywords: Pattern formation; Lamellar eutectic growth; Triple point; Directional solidification; Multiple variable expansion

1. Introduction

Eutectic growth during directional solidification of a binary mixture system consisting of species A and B has been a classic and fundamental subject in condensed matter physics and material science [1–15]. The typical experimental device used for studying such non-linear phenomena is the Hele-Shaw cell as shown in Fig. 1. The system consists of a thin sample material and two uniform temperature zones separated by a distance $(L)_D$: the hot zone with a temperature T_H higher than the eutectic temperature T_e; and the cold zone with a temperature T_C lower than the eutectic temperature T_e. The sample is pulled at a constant speed V along the direction from the hot zone to cold zone. We denote the concentration of species (B) in the mixture by C. In eutectic growth, the concentration C in the liquid state is moderate close to the eutectic concentration C_e. According to the phase diagram (see Fig. 2), due to the phase transition, a liquid state of the mixture is separated into two different solid phases as shown in Fig. 3: (i) the α-phase, in which the species (A) is the major component, while the species (B) is the minor component; and (ii) the β-phase, in which the species (B) is the major component, while (A) is the minor component. One of the prominent features of the eutectic interfacial pattern is that it has an triple point, where the angles between the interfaces are determined as the thermodynamic properties of the system. Furthermore, the experimental observations show that the system of steady eutectic growth may display the

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http://dx.doi.org/10.1016/j.actamat.2014.06.047
1359-6454/

Nomenclature

Constants

$\mathrm{Pe} = \frac{\ell_w}{\ell_D}$: the Peclet number, where ℓ_w is a half of interlamellar spacing.

$\lambda_T = \frac{\ell_D}{\ell_T}$: the ratio of the two diffusion lengths, ℓ_D and ℓ_T.

$\mathrm{G} = \frac{\ell_D}{\Delta H/(c_p\rho)}(G)_D$: the dimensionless gradient of the temperature, where $(G)_D = \frac{T_H - T_C}{(L)_D}$.

$W_c = \frac{w_c}{\ell_w}$: the dimensionless width parameter of the α-phase, where w_c is location of triple point.

$C_\infty = \varpi \hat{C}_\infty$: the dimensionless concentration in the far field.

φ The tilt angle.

Piece-wise constants:

$\mathbb{M} = -\frac{mC_e}{\Delta H/(c_p\rho)}$: the morphological parameter, where $m = (m_\alpha, m_\beta)$ is the slope of the liquidus.

$\Gamma = \frac{\ell_c}{\ell_w} = \frac{\ell_c \ell_D}{\ell_w^2}\frac{\ell_w}{\ell_D}$: the interfacial stability parameter, where $\ell_c = \frac{\gamma c_p \rho T_{M0}}{(\Delta H)^2}$, is the capillary length.

$R_c = \Gamma_\alpha / \Gamma_\beta$.

$\lambda_G = \frac{\ell_D}{\ell_G} = -\frac{\kappa_D}{V}\frac{(G)_D}{mC_e} = \frac{\mathrm{G}}{\mathbb{M}}$.

Unknowns

$C(x,z,t,\epsilon); h(x,t,\epsilon); z_*(t), W_c(t)$: the general unsteady concentration field, interface shape, location with eutectic temperature, location of triple point.

$C_B(x,z,\epsilon); h_B(x,\epsilon)$: the steady basic state solution.

$\{\bar{C}, \overline{C}_{00}, \overline{C}_{01}, \overline{C}_{10}, \ldots\}; \{z_*, z_{01}, \ldots\}$; $\{W_c, w_0, w_{01}, \ldots\}$; $\{\bar{h}, \bar{h}_{01}, \bar{h}_{10}, \ldots\}$: the outer solution and the outer expansions.

$\mathbb{h}(\hat{x},\epsilon); \{\overline{\mathbb{h}}_{01}(\hat{x}), \overline{\mathbb{h}}_{01}(\hat{x}), \ldots\}$: the inner solution for interface shape and inner expansion.

$\{\mathbb{h}_{*c}, z_{*c}\}$: the leading order approximation of composite solution for interface shape.

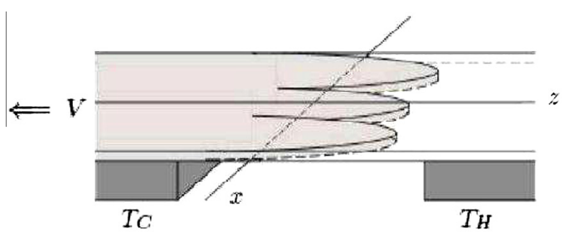

Fig. 1. A sketch of directional solidification device – Hele-Shaw cell.

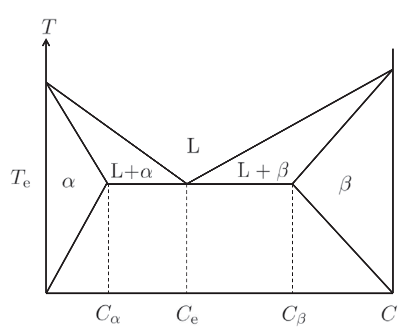

Fig. 2. A sketch of phase diagram of eutectic growth.

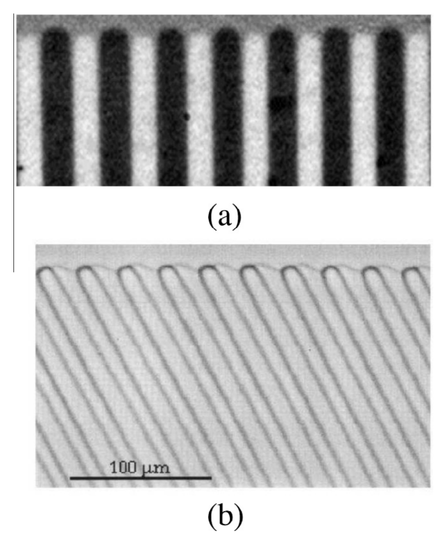

Fig. 3. (a) A typical steady eutectic growth with α–β interface parallel to z axis [14]. (b) A typical steady tilted eutectic growth. [9]

spatially-periodic, interfacial patterns, the patterns may be non-tilted or tilted as seen in Fig. 3(a) or (b), respectively. The early analytical theory of steady eutectic growth was established by Jackson and Hunt [1]. In the J–H theory, the steady state is described by a solution with *one free parameter* depending on the interlamellar spacing, and the interfaces are assumed being flat. As a consequence, in J–H's solution the effect of the triple point is totally neglected. Along with such assumptions, as a hypothesis J–H also proposed that the interlamellar spacing ℓ_w was selected by the minimum of average undercooling of the growth front. It has been long recognized that the J–H solution was not self-consistent, as it treated the related free boundary problem as a boundary value problem, whereas the sharp selection hypothesis proposed by J–H were also not well supported by the experimental data. The first work treating the eutectic growth as a free boundary problem was done by Nash in 1977 [4]. Nash derived a set of non-linear integro-differential equations for the system and solved it numerically for the simplified version