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Analysis of attitude motion evolutions of variable mass gyrostats and coaxial rigid bodies system

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ABSTRACT

This work involves the research into angular motion of variable mass gyrostats, coaxial bodies systems and dual-spin spacecraft in a translating coordinate frame. The variability of mass–inertia parameters of coaxial bodies causes non-trivial changes of system angular motion. The article describes qualitative method for phase space analysis, based on the evaluation of a phase trajectory curvature. The method can be used to investigate the phase trajectory shape and to synthesize conditions for special motion modes realization (for example, monotone decreasing/increasing of nutation angle). The paper results can be used to describe the motion of a variable mass dual-spin spacecraft, performing active maneuvers.

1. Introduction

Research into attitude motion of a system of coaxial rigid bodies and gyrostats always was and still remains one of the important problems of theoretical and applied mechanics. The dynamics of the attitude (angular) motion of rigid bodies, gyroscopes and gyrostats is classical topic of mechanics. Basic aspects of such motion were studied by Euler, Poinsot, Lagrange, Kovalevskaya, Zhukovsky, Volterra, Wangerin, Wittenburg [1–5].

However, the study of the dynamics of rigid bodies and gyrostats is still very important in modern theoretical, applied and space-flight mechanics. Among the basic directions of modern research within the framework of the indicated problem it is possible to highlight the following points: deriving exact and approximated analytical and asymptotic solutions [1–5,25,28], investigation of a stability of motion conditions [6–16], the analysis of motion under the influence of external regular and stochastic disturbance, research into dynamic chaos [17–22], study of non-autonomous systems with variable parameters [23–30].

Zhukovsky studied the motion of a rigid body containing cavities filled with homogeneous capillary liquid. The research showed that the equations of motion in such case could be reduced to the equations of the attitude motion of a gyrostat. Also analytical solutions of some special modes of gyrostat motion were found.

Assuming constant angular momentum Volterra analytically solved equations of gyrostat attitude motion. Volterra's solution has generalized a similar analytical solution in case of Euler for a rigid body. In the works of Wangerin and Wittenburg solution of Volterra is reduced to the convenient parameterization expressed in elliptic integrals.

Generalization of Lagrange's analytical solution for heavy gyrostat is given in a paper [25]. In this paper solution has been found in elliptic functions and integrals for all Euler's angles.

Attitude motion investigation results of gyrostats are very important for numerous space-flight applications. The attitude dynamics of gyrostat satellites and dual-spin (double-rotation) satellites has been studied by a number of scientists [6–22]. Most of these efforts were aimed on finding the equilibrium states in the presence of external disturbance torques [6–9], on analysis of the stability of spinning satellites under energy dissipation [10–16]. Some authors recently have investigated bifurcation and chaos in the gyrostat satellites [17–22].

Despite above-mentioned wide spectrum of research results the stated problem still remains actual, especially for the variable structure (mass) rigid bodies systems, gyrostats and dual-spin spacecraft (SC) with jet engine.

Any SC in orbit is affected by external disturbances of different kind, e.g. the solar radiation pressure, the gravity gradient torque,

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the magnetic torque caused by the Earth's magnetic field, or the aerodynamic torque due to the action of a resisting medium like planets atmosphere. However, all these external disturbances are not large in comparison with the jet engine thrust of the SC on the active motion stage (e.g. inter-orbital transfer, orbit correction, attitude reorientation). Moreover, variability of mass parameters (mass and moments of inertia) has a considerable influence on attitude dynamics. The change of the moments of inertia entails change of angular momentum, which is the basic characteristic of attitude motion. Thereupon mass (structure) variation is one of the primary factors determining attitude motion of a SC.

For the purposes of better understanding the essence of this problem it is important to give a brief overview of the main considered engineering peculiarities of SC's active motion. In order to perform an active maneuver (e.g. inter-orbital transfer) SC should create a jet engine thrust and thus obtain acceleration or braking momentum $\Delta\mathbf{V}$ (reorbit/ deorbit burn).

This momentum should be generated exactly in a pre-calculated direction. Engine thrust is usually focused along the SC's longitudinal axis; therefore it is necessary to stabilize the longitudinal axis in order to ensure the accurate momentum generation. Stabilization of the longitudinal axis can be carried out in a gyroscopic mode when SC spins around the longitudinal axis, which is oriented in the calculated direction.

Momentum generation is not instantaneous; it demands a continuous operation of the jet engine within several seconds (or minutes). During this period of time a SC performs two motions: trajectory motion of a center of mass and an angular motion around it. Such angular motion obviously changes the location of the longitudinal axis and, hence, a direction of thrust.

The time history of thrust direction strongly affects the value and direction of a transfer momentum deviation. Consequently, the transfer is performed to the orbit different from the desired one. There is a "scattering" of thrust (Fig. 1). Therefore, it is very important to take SC angular motion into account during the analysis of the powered trajectory motion.

It is necessary to obtain the angular motion which ensures that SC's longitudinal axis (and the thrust vector) performs precessional motion with monotonously decreasing nutation angle. Thus the longitudinal axis travels inside an initial cone of nutation and the thrust vector naturally comes nearer to an axis of a precession which is a desired direction of transitional momentum output ("is focused" along a necessary direction).

When the angular motion does not provide a monotonous decrease in nutation angle the longitudinal axis moves in a complicated way. In such case the thrust vector also performs complicated motion and "scatters" the transitional momentum. A transfer orbit scatters as well.

Among the works devoted to the rigid bodies systems with variable mass and inertia parameters it is possible to mark the following [17,18,23,24,28,30]. The work [18] contains the analysis of chaotic behavior of a spacecraft with a double rotation and time-dependent moments of inertia during its free motion. The main investigation results of variable mass system dynamics should be found in the monographies [23,24]. These results include Ivan V. Meschersky theory of motion of bodies with variable mass, theory of "short-range interaction" and "solidification (freezing)".

The equations of variable mass dynamically symmetrical coaxial bodies system were developed in papers [28]. Also in [27] the attitude motion of coaxial bodies system and double rotation spacecraft with linear time-dependent moments of inertia were analyzed and conditions of motion with decreasing value of nutation were found. The results [27] can be used for the analysis of attitude motion of a dual-spin spacecraft with an active solid-propellant rocket engine.

Current paper represents continuation of the research described in [27–30] and is devoted to the dynamics of variable mass coaxial bodies systems, unbalanced gyrostats and dual-spin spacecraft.

The paper has the following structure: Section 1—introduction of the primary theoretical and physical background, Section 2—mathematical definition of the coaxial bodies attitude motion problem in terms of the angular momentum, Section 3—main equations of attitude motion of two variable mass coaxial bodies system and unbalanced gyrostat, Section 4—development of research method for the attitude motion of variable mass coaxial bodies and unbalanced gyrostat, Section 5—examples of analysis of the attitude motion of variable mass unbalanced gyrostat, Section 6—conclusion.

2. Problem definition

Below we will develop a mathematical model of angular motion of a system of k coaxial rigid bodies of variable mass with respect to translating coordinate frame $OXYZ$.

The motion of the system is analyzed with respect to the following coordinate frames (Fig. 2): $P\xi\eta\zeta$ is fixed in absolute space frame; $OXYZ$ is a moving (non-inertial) coordinate frame, the axes of which remain parallel with the axes of the fixed frame during the whole time of motion; $Ox_iy_iz_i$ are bodies frames connected to coaxial bodies ($i=1,2,\ldots,k$). $OXYZ$ system has its origin in a point lying on the common axis of rotation of the bodies and matching with the initial position of the center of mass ($t=t_0 : C\equiv O$). Points in the different parts of the system are distinguished by the body they belong to, and in all expressions they are indicated by the subscript ν_i (where i is a number of an appropriate body).

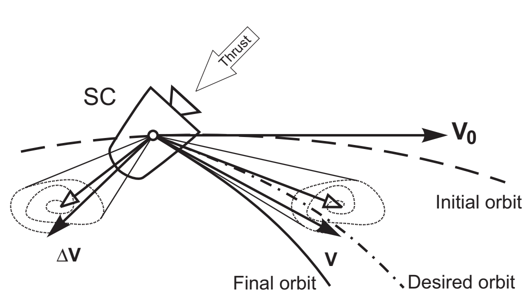

Fig. 1. Scattering of thrust and transfer orbit.

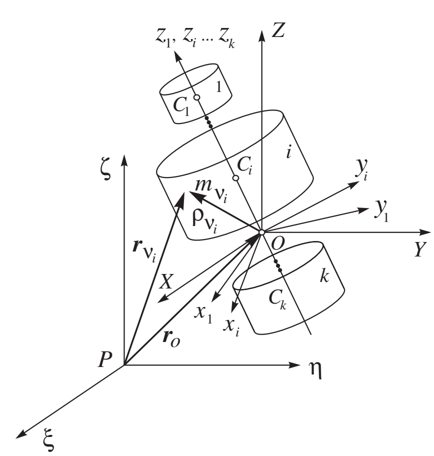

Fig. 2. Coaxial bodies system and coordinate frames.