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International Journal of Pressure Vessels and Piping 83 (2006) 1–12
INTERNATIONAL JOURNAL OF Pressure Vessels and Piping
www.elsevier.com/locate/ijpvp

An efficient method for free vibration analysis of rotating truncated conical shells

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Received 15 July 2005; received in revised form 24 October 2005; accepted 24 October 2005

Abstract

This paper proposes a discrete singular convolution method for the free vibration analysis of rotating conical shells. A regularized Shannon's delta kernel is selected as the singular convolution to illustrate the present algorithm. Frequency parameters of the forward modes are obtained for different types of boundary conditions, rotating velocity and geometric parameters. The present results compare well with numerical data available in the literature. Numerical experiments reveal that the present approach is accurate, efficient and reliable for vibration analysis of conical shells.

Keywords: Frequencies; Discrete singular convolution; Rotating conical shells; Vibration

1. Introduction

Circular and conical shell structures are increasingly being used in many engineering applications. As a consequence, the vibration of shell structures has been extensively studied [1–3]. Rotating conical shells are being widely used in the drive shafts of gas turbines, high-speed centrifugal separators, high-power aircraft jet engine, motors and rotor systems. A number of analytical and numerical studies have been conducted on the vibration analysis of rotating conical shells [4–6]. Chun and Bert [7] studied rotating composite cylindrical shells using some thin shell theories. Chen et al. [8] applied the finite element method for rotating shells. Lam and Hua [9,10] and Hua and Lam [11] have analyzed the free vibration of rotating circular conical shells. The influence of boundary conditions on the frequency characteristics of a rotating truncated circular conical shell has been examined by Lam and Hua [12,13]. A few studies concerning free vibration analysis of conical shells have been carried out, namely by Liew and Lim [14], Liew et al. [15,16], Lim and Liew [17,18], Lim et al. [19], Shu [20], and Tong [21].

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0308-0161/$ - see front matter
doi:10.1016/j.ijpvp.2005.10.005

2. Theoretical formulation

Consider a truncated circular conical shell rotating about its symmetrical and horizontal axis at an angular velocity ω as shown in Fig. 1. The cone semivertex angle, thickness of the shell, and cone length are denoted by α, h and L, respectively. R_1 and R_2 are the radii of the cone at its small and large edges. The conical shell is referred to a coordinate system (x,θ,z) as shown in Fig. 1. The components of the deformation of the conical shell with reference to this coordinate system are denoted by u, v, w in the x, θ and z directions, respectively. The equilibrium equation of motion in terms of the force and moment resultants can be written as

$$L_x(u,v,w)-\rho_t\frac{\partial^2 u}{\partial t^2}=0, \tag{1a}$$

$$L_\theta(u,v,w)-\rho_t\frac{\partial^2 v}{\partial t^2}=0, \tag{1b}$$

$$L_z(u,v,w)-\rho_t\frac{\partial^2 w}{\partial t^2}=0, \tag{1c}$$

where

$$L_x=\frac{\partial N_x}{\partial x}+\frac{\sin\alpha}{R(x)}(N_x-N_\theta)+\frac{1}{R(x)}\frac{\partial N_{x\theta}}{\partial\theta}$$
$$+\rho h\omega^2\left[\frac{\partial^2 u}{\partial\theta^2}-r\cos\alpha\frac{\partial w}{\partial x}\right]+2\rho h\omega\sin\alpha\frac{\partial v}{dt}, \tag{2}$$

Nomenclature

E, ν	modulus of elasticity and Poisson's ratio
h, L, R	thickness, length and radius of conical shell
R_1, R_2	radii of the cone at small and large edges
ρ, ρ_t	density and average density in the z direction
A_{ij}, B_{ij}, D_{ij}	extensional, coupling and bending stiffnesses
x, θ, z	coordinates in meridional, circumferential and normal directions
u, v, w	displacements in meridional, circumferential and normal directions
ε_i, κ_i	reference strains and reference curvatures
N_i, M_i	resultant force and moment
Ω	non-dimensional frequency parameter
ϖ	frequency parameter
ω	angular velocity
t	time
n	circumferential wave number
L_x, L_θ, L_z	differential operators defined by Eqs. (2)–(4)
Q_x, Q_θ	transverse shear force resultants

$$L_\theta = \frac{\partial N_{x\theta}}{\partial x} + \frac{1}{R(x)}\frac{\partial N_\theta}{\partial \theta} + \frac{2\sin\alpha}{R(x)}N_{x\theta} + \frac{\cos\alpha}{R(x)}\frac{\partial M_{x\theta}}{\partial x} + \frac{\cos\alpha}{R^2(x)}\frac{\partial M_\theta}{\partial \theta} + \rho h\omega^2\left[R(x)\frac{\partial^2 u}{\partial x\partial\theta} + R(x)\sin\alpha\frac{\partial v}{\partial x} + \sin\alpha\frac{\partial u}{\partial\theta}\right] - 2\rho h\omega\left[\sin\alpha\frac{\partial u}{\partial t} + \cos\alpha\frac{\partial w}{\partial t}\right], \tag{3}$$

$$L_z = \frac{\partial^2 M_x}{\partial x^2} + \frac{2}{R(x)}\frac{\partial^2 M_{x\theta}}{\partial x\partial\theta} + \frac{1}{R^2(x)}\frac{\partial^2 M_\theta}{\partial\theta^2} + \frac{2\sin\alpha}{R(x)}\frac{\partial M_x}{\partial x} - \frac{\sin\alpha}{R(x)}\frac{\partial M_\theta}{\partial x} + \frac{\cos\alpha}{R(x)}N_\theta + \rho h\omega^2\left[R(x)\frac{\partial^2 w}{\partial\theta^2} - R(x)\cos\alpha\frac{\partial u}{\partial x} + w\cos^2\alpha + u\sin\alpha\cos\alpha\right] + 2\rho h\omega\left[\cos\alpha\frac{\partial v}{\partial t}\right], \tag{4}$$

where

$$R(x) = R_1 + x\sin\alpha \tag{5}$$

and

$$\rho_t(x,\theta) = \frac{1}{h}\int_{-h/2}^{h/2}\rho(x,\theta,z)\mathrm{d}z \tag{6}$$

where ρ and ρ_t are, respectively, the density and density per unit length. The transverse shear force resultants can be given from M_x, M_θ and $M_{x\theta}$ by

$$Q_x = \frac{1}{R(x)}\frac{\partial}{\partial x}[R(x)M_x] - \frac{M_\theta\sin\alpha}{R(x)} + \frac{1}{R(x)}\frac{\partial M_{x\theta}}{\partial\theta}, \tag{7}$$

$$Q_\theta = \frac{1}{R(x)}\frac{\partial}{\partial x}[R(x)M_{x\theta}] + \frac{M_{x\theta}\sin\alpha}{R(x)} + \frac{1}{R(x)}\frac{\partial M_\theta}{\partial\theta}. \tag{8}$$

Moment resultants and in-surface force can be obtained by

$$N = (N_x, N_\theta, N_{x\theta})^T = \int_{-h/2}^{h/2}(\sigma_x, \sigma_\theta, \sigma_{x\theta})^T\mathrm{d}z, \tag{9}$$

$$M = (M_x, M_\theta, M_{x\theta})^\mathrm{T} = \int_{-h/2}^{h/2}(\sigma_x, \sigma_\theta, \sigma_{x\theta})^\mathrm{T} z\,\mathrm{d}z, \tag{10}$$

where the stress vector field $(\sigma)^\mathrm{T} = \{\sigma_x, \sigma_\theta, \sigma_{x\theta}\}$. Based on Love's first approximation theory, the strain components of

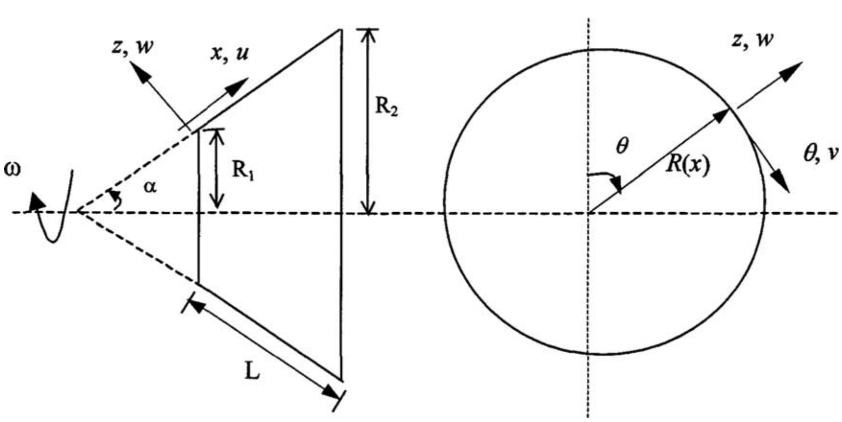

Fig. 1. Geometry and notation of rotating conical shell.