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A two-fluid model for pulsatile flow in catheterized blood vessels

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ABSTRACT

The pulsatile flow of blood through a catheterized artery is analyzed, assuming the blood as a two-fluid model with the suspension of all the erythrocytes in the core region as a Casson fluid and the peripheral region of plasma as a Newtonian fluid. The resulting non-linear implicit system of partial differential equations is solved using perturbation method. The expressions for shear stress, velocity, flow rate, wall shear stress and longitudinal impedance are obtained. The variations of these flow quantities with yield stress, catheter radius ratio, amplitude, pulsatile Reynolds number ratio and peripheral layer thickness are discussed. It is observed that the velocity distribution and flow rate decrease, while, the wall shear, width of the plug flow region and longitudinal impedance increase when the yield stress increases. It is also found that the velocity increases, but, the longitudinal impedance decreases when the thickness of the peripheral layer increases. The wall shear stress decreases non-linearly, while, the longitudinal impedance increases non-linearly when the catheter radius ratio increases. The estimates of the increase in the longitudinal impedance are considerably lower for the present two-fluid model than those of the single-fluid model.

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1. Introduction

Blood is a multi-component material which consists of gel-like formed elements in aqueous plasma: red blood cells (RBCs 98% by volume), and some white blood cells (WBCs), and platelets and a variety of lipoproteins. Plasma is an aqueous solution of various proteins, including clotting factors (fibrinogen, prothrombin, factor-VII, factor-XIII, etc.) and various ions [1]. RBCs are very numerous and morphologically very simple. They contain hemoglobin which transports oxygen around the body [2]. Henderson and Thurston [3] have reported that platelets are very small but extremely important in relation to blood coagulation both in the healing of wounds and in the formation of thrombi.

Under normal condition, blood circulates within the body's vascular network. However, it has an inherent tendency to clot that is balanced by endothelium. The clot formation occurs for various reasons: endothelial injury, endothelial dysfunction, or flow stagnation and recirculation among others. Clot formation occurs when the initiating stimulus exceeds certain threshold. Clots are formed at the end of a series of interacting biochemical processes: platelet adhesion, activation and aggregation, coagulation (extrinsic and intrinsic), polymerization of fibrin monomers formed from fibrinogen, and cross linking of the fibrin polymer strands to form a fibrin network. A detailed overview of the process of clot formation and lysis is given in Anand et al. [1,4]. Fogelson [5] formulated continuum models for platelet aggregation and analyzed its mechanical properties. Fogelson and Guy [6] extended these continuum models further to study the platelet–wall interactions of platelet thrombosis, using numerical solution.

Mann et al. [7,8] discussed extensively the models of blood coagulation and the dynamics of thrombin formation. Attaullakhanov et al. [9] have experimentally analyzed the spatio-temporal dynamics of blood coagulation and pattern formation. Panteleev et al. [10] developed mathematical models for the study of blood coagulation and platelet adhesion in their review and provided some clinical applications of the mathematical models. Lawson et al. [11] studied the complex-dependent inhibition of factor VIIa by antithrombin III and heparin. Lawson et al. [12] built an experimental model for the tissue factor pathway to thrombin.

As the seminal contribution to the study of shear thinning viscoelastic nature of blood, Thurston [13] developed an extended Maxwell model which is applicable to one-dimensional flow. Anand and Rajagopal [14] have studied extensively a shear-thinning viscoelastic fluid model for blood flow within a thermodynamic framework that takes cognizance of the fact that viscoelastic fluids can remain stress free in several configurations. Anand et al. [4] developed a mathematical model for the formation and lysis of blood

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clot. Anand et al. [1] built a viscoelastic model within the thermodynamic frame of reference for analyzing the mechanics of a coarse ligated plasma dot.

In modern medicine, with the evolution of coronary balloon angioplasty, there has been considerable increase in the use of catheters of various sizes. Coronary catheter probes are widely used as they provide valuable information about arterial anatomy and the hemodynamic significance of stenosis [15]. Catheter probes have also been used in conjunction with computational fluid dynamics to study the relationship between the flow patterns and atheroma formation [16], for which the accurate measurement of velocity is crucial. Typically, a catheter consists of a long flexible cylindrical tube at the tip of which various functional tools (e.g. pressure transducers, flow meters, inflatable balloons, etc.) are positioned. The purpose of catheters is to accurately measure the arterial pressure or pressure gradient, or to clear short occlusions from the walls of the stenosed artery [17]. The method of catheterization is to insert the catheter into a peripheral artery and then position the device into the desired part of the arterial network by passing an appropriate length of the catheter through the artery [18]. The insertion of a catheter into an artery leads to the formation of an annular region between the catheter wall and the arterial wall. The insertion of a catheter into an artery alters the flow field, modifies the pressure distribution and hence increases the resistance to flow [19].

Back [20] and Back et al. [21] have studied the important hemodynamic characteristics like the wall shear stress, pressure drop and frictional resistance in catheterized coronary arteries under the normal and pathological situation of a stenosis present. The effect of catheterization on various flow quantities in a curved artery is studied by Jayaraman and Tiwari [22]. Daripa and Dash [17] have performed the numerical study of pulsatile blood flow in an eccentric catheterized artery using a fast algorithm. Apart from the above investigations, some more attempts [23–25] have been made to study the blood flow through catheterized arteries, treating blood as a Newtonian fluid. But, blood being a suspension of erythrocytes; it exhibits remarkable non-Newtonian behavior when it flows through narrow blood vessels at low shear rates [26–28]. Sankar and Hemalatha [19] and Dash et al. [26] have estimated the increase in the resistance for the blood flow through catheterized arteries for steady and pulsatile flow by treating blood as a non-Newtonian fluid.

Casson [29] examined the validity of Casson fluid model in studies pertaining to the flow characteristics of blood and reported that at low shear rates the yield stress for blood is non-zero. Casson fluid model is a non-Newtonian fluid model with non-zero yield stress. It has been demonstrated by Scott Blair [30] and Copley [31] that the parameters appropriate to Casson fluid-viscosity, yield stress and power-law are adequate for the representation of the simple shear behavior of blood. It has been established by Merrill et al. [32] that Casson fluid model holds satisfactorily for blood flowing in tubes of diameter 130−1300 μm. Charm and Kurland [33] pointed out in their experimental findings that the Casson fluid model could be the best representative of blood and that it could be applied to human blood. Further, Scott Blair and Spanner [34] reported that blood behaves like a Casson fluid in the case of moderate shear rate flows.

It has been pointed out by Scott Blair [35] and Iida [36] that though it is possible to model the blood flow by both Casson fluid model and Herschel–Bulkley fluid over the range where both models are valid, Casson fluid model is well suited and simple to apply for blood flow problems. Thurston [37] pointed out that the viscoelastic nature of blood is less prominent with increasing shear rate. Thurston [38,39] has also observed that there exists a critical shear rate beyond which the assumptions of linear viscoelasticity and Newtonian behavior of blood cease to hold and related the non-linear behavior to the microstructural changes that occur in blood with increasing shear rate. Hence, it is appropriate to assume the non-Newtonian fluid characterizing the blood in the core region of the two-fluid blood flow model as Casson fluid rather than as Herschel–Bulkley fluid model or as a viscoelastic fluid model.

Many researchers have used Casson fluid model for mathematical modeling of blood flow through narrow arteries at low shear rates for different flow situations. Chaturani and Ponnalagar Samy [40] have analyzed the pulsatile flow of Casson fluid for blood flow through stenosed arteries using perturbation method. Dash et al. [26] have studied the steady and pulsatile flow of Casson fluid for blood flow through catheterized arteries using perturbation analysis and they have computed the increase in the resistance to flow due to catheterization and non-Newtonian effects.

Some researchers [41–43] propounded that for blood flowing through small vessels, there is erythrocyte-free plasma (Newtonian) layer adjacent to the vessel wall and a core layer of a suspension of all erythrocytes (non-Newtonian). Accepting this idea, several studies [43–47] revealed that the existence of the peripheral layer has some significance in the functioning of the flow characteristics in the arterial system. Hence, in this paper, we study the pulsatile flow of a two-fluid model for blood flow through catheterized narrow arteries (of diameters 0.02–0.2 mm) at low shear rates ($\dot{\gamma} < 10$/s), assuming the suspension of all the erythrocytes in the core region of the blood vessel as a Casson fluid and the plasma in the peripheral layer as a Newtonian fluid. The layout of the paper is as follows.

Section 2 formulates the model mathematically, while Section 3 non-dimensionalizes the basic governing equations and the boundary conditions. The resulting implicit system of non-linear partial differential equations is solved using perturbation method in Section 4. The effects of pulsatility, catheterization, non-Newtonian nature of blood and peripheral layer thickness on various flow quantities are analyzed with some possible applications in Section 5. The results are summarized, and, some scope and possible extension of the present study are mentioned in Section 6.

2. Formulation of governing equations

Consider an axially symmetric, pulsatile, laminar, and fully developed flow of blood in an artery of radius $\bar{R}$ in which a catheter of radius $k\bar{R}$ ($k < 1$) is introduced coaxially and the blood is modeled as a two-fluid model with the suspension of all the erythrocytes in the core region as a Casson fluid and the plasma in the peripheral region as a Newtonian fluid. It is assumed the pulsatile flow in the artery is due to a prescribed periodic pressure gradient along the axis of the artery. The length of the artery is assumed to be large enough when compared to its diameter so that the entrance, end and special wall effects can be neglected. The cylindrical polar coordinate system $(\bar{r}, \bar{\phi}, \bar{z})$ is used to study the flow, where $\bar{r}$ and $\bar{z}$ denote the radial and axial coordinates and $\bar{\phi}$ is the azimuthal angle. The geometry of the catheterized artery is shown in Fig. 1. It can be shown that the radial velocity is negligibly small in magnitude and may be neglected for low Reynolds number flow. The basic momentum equations in this case simplifies to

$$\bar{\rho}_C \frac{\partial \bar{u}_C}{\partial \bar{t}} = -\frac{\partial \bar{p}}{\partial \bar{z}} - \frac{1}{\bar{r}} \frac{\partial}{\partial \bar{r}}(\bar{r}\bar{\tau}_C) \quad \text{if } k\bar{R} \leqslant \bar{r} \leqslant \bar{R}_1 \tag{1}$$

$$\bar{\rho}_N \frac{\partial \bar{u}_N}{\partial \bar{t}} = -\frac{\partial \bar{p}}{\partial \bar{z}} - \frac{1}{\bar{r}} \frac{\partial}{\partial \bar{r}}(\bar{r}\bar{\tau}_N) \quad \text{if } \bar{R}_1 \leqslant \bar{r} \leqslant \bar{R} \tag{2}$$

where $\bar{p}$ denotes the pressure; $\bar{\rho}_C$ and $\bar{\rho}_N$ denote the density of the Casson fluid and Newtonian fluid, respectively; $\bar{\tau}_C$ and $\bar{\tau}_N$ denote the shear stress of the Casson fluid and Newtonian fluid, respectively; $\bar{u}_C$ and $\bar{u}_N$ denote the fluid's velocity in the core region and peripheral region, respectively; $\bar{t}$ denotes the time and $\bar{R}_1$ is the radius of the core region of the artery. The relations between the shear stress and strain rate of the fluid in motion in the core region (Casson fluid)

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