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A new method for characterizing patterns of neural spike trains and its application

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A B S T R A C T

A method for characterizing and identifying firing patterns of neural spike trains is presented. Based on the time evolution of a neural spike train, the counting process is constructed as a time-dependent stair-like function. Three characteristic variables defined at sequential moments, including two formal derivatives and the integration of the counting process, are introduced to reflect the temporal patterns of a spike train. The reconstruction of a spike train with these variables verify the validity of this method. And a model of cold receptor is used as an example to investigate the temporal patterns under different temperature conditions. The most important contribution of our method is that it not only can reflect the features of spike train patterns clearly by using their geometrical properties, but also it can reflect the trait of time, especially the change of bursting of spike train. So it is a useful complementarity to conventional method of averaging.

1. Introduction

It was well known that neural information processing relies on the transmission of a series of stereotyped events called action potentials or spikes. Temporal recording of firing events of a neuron provides an inter-spike-interval (*ISI*) series. It is expected that the processed neural information can be encoded in the structure of *ISI* series, that is, the neural firing activities can be represented by the patterns of neural spike trains. The basic biophysics underlying the generation of action potentials (spikes) is somewhat well established, but the encoding mechanism is still unclear.

At present, there are many existing methods to explore how neural information respond to different stimuli. Many researchers measured the statistical significance of temporal structures in spike trains in order to determine how much information about stimuli contained in neural responses by means of the information theory [1,2]. For example, to investigate the encoding meaning of spike timing and the temporal rhythm structures (that is, patterns) of spikes, the series expansion approximation method [3], the information distortion method [4] and other methods have also been used to quantify the information encoded in spike trains [5–10]. These works indicated that the recognition of temporal patterns of a spike train is essential for extracting information from neuronal responses; however, the features of conveyed information have not been well understood yet. Therefore, it is important to develop some other methods to recognize the temporal patterns of spike trains, that is, to understand the information represented by the neuron. Several characteristic variables were proposed to quantify the structure of spike trains [11], but their geometric meaning and the use in the reconstruction of spike trains are still not clear.

Taking into account of the above situation, an approach to identify the temporal structures of spikes in neural systems is developed in this paper. The key feature of this method is to introduce the counting process, which exhibits the dynamic characteristic variables of spike trains. Several characteristics of spike train are deduced from the counting process at sequential moments and used to describe the temporal pattern of a spike train. The comparison with the usual tuning curve method is also developed.

This paper is organized as follows. The counting process with its formal derivatives and integration are introduced to characterize the temporal pattern of a spike train in Section 2. An example for the response of a cold receptor and the comparison with previous methods are presented in Section 3, and the conclusion is given in Section 4.

2. Methodology and counting process

2.1. Counting process and its first formal derivative

A recorded neural spike train can be characterized by a bounded variation function $q(t)$ called counting process as follows:

$$q(t) = i, \quad t_i \leqslant t < t_{i+1} \quad (i = 1, 2, \ldots), \tag{1}$$

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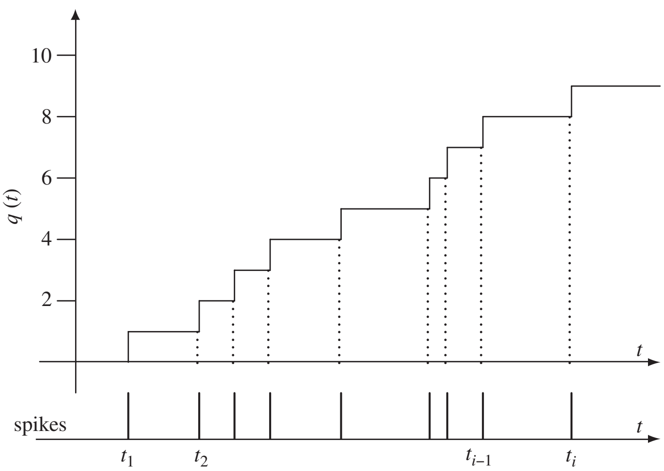

Fig. 1. Counting process of a spike train.

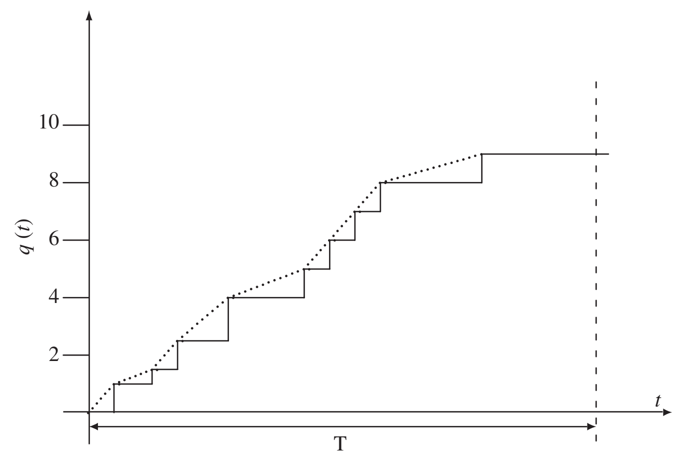

Fig. 2. Illustration of the instantaneous firing rate.

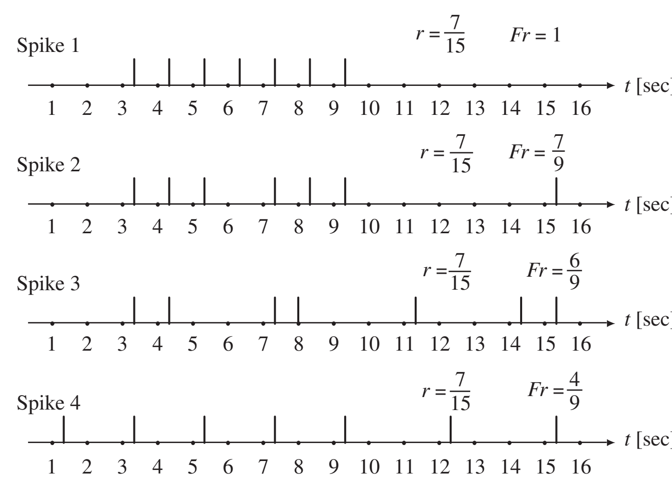

Fig. 3. Spike patterns with the same r, but different Fr. Here $q_j = 7$, $ISI_{min} = 1$, $T = 15ISI_{min}$.

where t_i is the firing moment of the i-th spike. That is to say, $q(t)$ is the counting number of spikes firing before time t, which is a monotonic bounded variation function of time as shown in Fig. 1. It can completely reflect the temporal rhythm structures (or patterns) of spikes and the information of stimuli. To relate the response to stimuli, Newton's method is adopted to investigate the first and second derivatives of the counting process. It is motivated by the well-known fact that a smooth function can be well depicted by its derivatives through Taylor's expansion. Geometrically, one can approximate a piece of curve by its slope and curvature around a point.

It is known that if there are N_0 spikes within a spike train over a time bin T, the traditional firing rate can be generally defined by

$$r = \frac{N_0}{T}. \tag{2}$$

Taking account of the counting process, the firing rate over the time interval Δt at time t can be taken as

$$Fr(t) = \frac{\Delta q(t)}{\Delta t}, \tag{3}$$

that is, its first order difference of the counting process over Δt. Especially, for the i-th spike, we take $\Delta t = ISI(i)$ (that is, the time interval between the i-th and the $i+1$-th spikes) with a step $\Delta q(t)=1$, and then (3) can be rewritten in the following form:

$$Fr(i) = \frac{1}{ISI(i)}, \tag{4}$$

which is called the instantaneous firing rate at the i-th spike. This can be demonstrated by the slopes of ordered segments shown as dotted lines in Fig. 2.

Averaging $Fr(i)$ over a time bin T leads to the averaged firing rate

$$Fr = \frac{1}{N_0 - 1} \sum_{i=1}^{N_0-1} \frac{1}{ISI(i)}, \tag{5}$$

where N_0 is the total number of spikes in this time bin T. Here Fr is called the first formal derivative of the counting process with dimension 1/s. Phenomenally, Fr measures the averaged density of spikes over the interval [0, T]. Using both the firing rate (2) and the first formal derivative (5), we can identify more information about spike patterns. Some examples are shown in Fig. 3, where all spike trains have the same firing rate $r = 7/15$ but different Fr within this time bin. It is seen that the greater Fr is, the closer the spikes gather together, and vice versa.

2.2. Second formal derivative of counting process

In order to explore more information of the temporal patterns of spikes, we introduce the second formal derivative of the counting process as follows:

$$Sr(t) = \frac{Fr(t + \Delta t) - Fr(t)}{\Delta t}. \tag{6}$$

Similar to (4), the instantaneous value of $Sr(t)$ at the i-th spike can be defined as

$$Sr(i) = \frac{Fr(i+1) - Fr(i)}{2min[ISI(i), ISI(i+1)]}, \tag{7}$$

where $\min[ISI(i), ISI(i+1)]$ is the minimum of $ISI(i)$ and $ISI(i+1)$. Geometrically, it represents the 'curvature' of the counting process curve. The 'convexity' or 'concavity' of the response curve, determined by the sign of $Sr(i)$, reflects the variation tendency of $ISIs$ shown in Fig. 4.

Averaging over a time bin T, we have

$$Sr = \frac{1}{N_0 - 1} \sum_{i=1}^{N_0-1} \frac{Fr(i+1) - Fr(i)}{2min[ISI(i), ISI(i+1)]}, \tag{8}$$

which is called the second formal derivative of the counting process, characterized a spike train by the variation tendency of $ISIs$. The above three quantities, that is, the firing rate (2) and two formal