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Direct method for analysis of flexible cantilever beam subjected to two follower forces

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ABSTRACT

The static analysis of the flexible non-uniform cantilever beams under a tip-concentrated and intermediate follower forces is considered. The angles of inclination of the concentrated forces with respect to the deformed axis of the beam remain unchanged during deformation. The governing non-linear boundary-value problem is reduced to an initial-value problem by change of variables. The resulting problem can be solved without iterations. It is shown that there are no critical loads in the Euler sense (divergence) for any flexural–stiffness distribution and angles of inclination of the follower forces. In particular, if the follower forces are tangential, the rectilinear shape of the non-uniform cantilever beam is the only possible equilibrium configuration. In this paper some equilibrium configurations of the uniform cantilever under normal or tangential follower forces are presented using direct method.

1. Introduction

There are many research studies dealing with large-deflection problem of a cantilever beam subjected to a follower forces. Argyris and Symeonidis in their fundamental paper [1] performed static geometrically non-linear analysis of cantilevers subjected to follower loads by the finite-element method and found the critical flutter loads. The finite difference method has been applied in solutions for the large bending of uniform cantilever subjected to concentrated or distributed follower loads by Saje and Srpcic [2]. When follower loads are tip-concentrated (normal or tangential to the deformed axis of the cantilever) this method leads to a system of transcendental equations which can be solved without iteration [2]. Rao et al. [3,4] studied large deflections of uniform and non-uniform cantilever beams under tip rotational loads using the shooting method. In particular, the case when the force at the free end maintains a constant angle with the beam axis was considered. The large deflections and stability behavior of cantilever beams subjected to transverse follower force (using the finite-element method) was studied by Vitaliani et al. [5]. Detinko [6] presented the closed analytic solution of the large-deflection problem for cantilever beams and circular arches of uniform cross section, subjected to terminal follower forces. The elastica solutions for an uniform cantilever beam under two proportional follower forces normal to the deformed beam axis was obtained with the help of the principle of elastic similarity by Hartono [7].

In Ref. [8], the direct numerical method for the large-deflection problem of a non-uniform cantilever under a tip-concentrated follower force was proposed. It is shown that there are no static critical loads (divergence) for any flexural–stiffness distribution and angles of inclination of the follower force. It is of interest to assess the validity of this statement for the cantilever under a few follower forces.

In the present paper, the large-deflection problem of a non-uniform cantilever beam under two concentrated follower forces is considered. The angles of inclination of the forces with respect to the deformed axis of the beam are assumed to be constant. The mathematical formulation of this problem yields a non-linear two-point boundary-value problem which is reduced to an initial-value problem by change of variables. The advantage of this approach is that the problem can be solved without iterations. Since the solution of the initial-value problem is unique, divergence instability does not occur. Therefore, the elastic cantilever beam in question can lose stability only by flutter. In particular, if the follower forces are tangential, the rectilinear shape of the non-uniform cantilever beam is the only possible equilibrium configuration. Some equilibrium configurations of a uniform cantilever under normal or tangential follower forces are presented.

2. Formulation of the problem

Let us consider a rectilinear non-uniform cantilever beam having length L and flexural rigidity $EI(s)$ subjected to two concentrated follower forces P_1, P_2. The force P_1 is applied to the free end of cantilever, while P_2 is applied at a distance μL from the free end (Fig. 1). The angles of inclination of the forces with respect to the deformed axis of the beam α_1, α_2 are kept constant. The arc length measured

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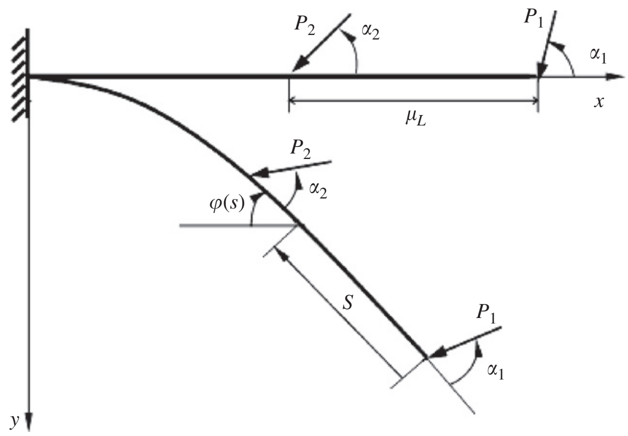

Fig. 1. Cantilever beam under two follower forces.

from the free end and the slope of the centroidal axis of the beam are denoted by s and $\varphi(s)$, respectively. Using the Euler–Bernoulli law of bending states, the non-linear differential equation governing the behavior of the beam can be obtained

$$(EI\varphi')' + P_1 \sin(\varphi + \alpha_1 - \varphi(0)) = 0 \quad \text{for } s \in [0,\ \mu L)$$
$$(EI\varphi')' + P_2 \sin(\varphi + \alpha_2 - \varphi(\mu L))$$
$$+ P_1 \sin(\varphi + \alpha_1 - \varphi(0)) = 0 \quad \text{for } s \in [\mu L,\ L] \tag{1}$$

with the boundary conditions

$$\varphi'(0) = 0, \quad \varphi(L) = 0. \tag{2}$$

The angles $\alpha_1 = \alpha_2 = \pi/2$ correspond to the follower forces acting in the normal direction to the deformed axis of the beam [1–8] and the angles $\alpha_1 = \alpha_2 = 0$ correspond to the tangential follower forces [2,8].

Once the slope $\varphi(s)$ has been found, the Cartesian coordinates of the beam axis are readily determined from the relations

$$x(s) = \int_s^L \cos\varphi\, d\tilde{s}, \quad y(s) = \int_s^L \sin\varphi\, d\tilde{s}. \tag{3}$$

3. Method of solution

Non-linear two-point boundary-value problems similar to that formulated above are usually solved by iterative methods. According to the shooting method, the non-linear two-point boundary-value problem (1), (2) can be reduced to a set of initial-value problems and the unknown initial value is then determined iteratively [3,4]. It is well known that the convergence of the iterative procedure depends upon the proximity of the initial guess to the particular solution sought. Moreover, similar boundary-value problems for conservative problems (the flexible cantilever beam subjected to inclined dead forces) admit multiple equilibrium solutions [9].

It can be shown, however, that the problem formulated above can be solved by direct method without iterations.

Let us introduce a new variable [8]:

$$z(s) = \varphi(s) + \alpha_1 - \varphi(0). \tag{4}$$

As a result, the boundary-value problem (1), (2) is reduced to the initial-value problem

$$(EIz')' + P_1 \sin(z) = 0 \quad \text{for } s \in [0, \mu L),$$
$$(EIz')' + P_2 \sin(z + \alpha_2 - z(\mu L))$$
$$+ P_1 \sin(z) = 0 \quad \text{for } s \in [\mu L, L], \tag{5}$$

$$z(0) = \alpha_1,\ z'(0) = 0 \tag{6}$$

with the supplementary condition

$$z(L) = \alpha_1 - \varphi(0). \tag{7}$$

Introducing the notation

$$z_1 = z, \quad z_2 = EI(s)z', \tag{8}$$

the problem (5), (6) can be reduced to the normal system of non-linear differential equations

$$z_1' = z_2/EI(s),$$
$$z_2' = -P_1 \sin(z_1) \quad \text{for } s \in [0,\ \mu L),$$
$$z_2' = -P_2 \sin(z_1 + \alpha_2 - z_1(\mu L))$$
$$- P_1 \sin(z_1) \quad \text{for } s \in [\mu L,\ L], \tag{9}$$

$$z_1(0) = \alpha_1, \quad z_2(0) = 0. \tag{10}$$

System (9), (10) can be integrated over a given interval $s \in [0, L]$ by a standard numerical method. From Eq. (7) the value of the tip slope of the beam

$$\varphi(0) = \alpha_1 - z_1(L) \tag{11}$$

and values of $\varphi(s)$ are calculated by the formula

$$\varphi(s) = z_1(s) - z_1(L), \quad s \in [0, L], \tag{12}$$

which follows from Eq. (4).

Thus, in contrast to the shooting method the problem considered is solved without iterations.

4. Analysis and results

The solution of initial-value problem (5), (6) is unique for a continuous function $EI(s)$ and any fixed values of $P_1,\ P_2,\ \alpha_1,\ \alpha_2,\ \mu$. If the follower forces are tangential ($\alpha_1 = \alpha_2 = 0$), the problem has a unique solution $z(s) = \varphi(s) \equiv 0$, which means that the straight configuration is the only equilibrium configuration of the beam. Therefore, the considered cantilever beams have no critical loads in the Euler sense (divergence) for any flexural–stiffness distributions along the beam. It follows that the non-uniform cantilever beam in question can exhibit only dynamic instability (flutter) [1,5,10].

These conclusions generalize the same results for non-uniform cantilever beams under a tip-concentrated follower force [8]. Thus, in contrast to the conservative systems [9], the studied non-conservative systems always have a unique solution (equilibrium configuration) that can be found by direct method.

Using the method of solution outlined above, the behavior of a uniform cantilever subjected to a tip and intermediate follower

Table 1
Tip coordinates and slope of a cantilever loaded by normal follower forces ($P_1 = P_2 = P,\ \alpha_1 = \alpha_2 = \pi/2,\ \mu = 0.5,\ \tilde{P} = PL^2/EI$).

$\tilde{P}$	$\varphi(0)$	$x(0)/L$	$y(0)/L$
1	35.43	0.8935	0.4124
3	98.37	0.3240	0.8148
5	144.84	−0.1177	0.7208
10	204.58	−0.2922	0.3194
12	216.09	−0.2506	0.2477
15	227.60	−0.1810	0.1896
20	240.11	−0.0863	0.1506
25	249.45	−0.0223	0.1466