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A note on internal damping induced self-excited vibration in a rotor by considering source loading of a DC motor drive

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ABSTRACT

It is well known that rotors become unstable beyond a certain threshold spinning speed due to the non-conservative circulatory forces, which arise out of rotating internal damping. In this note, it is shown that if the source loading of the non-ideal drive is considered then this instability manifests itself as a constant rotor spinning speed and a constant amplitude whirl orbit about an unstable equilibrium. A DC motor drive is considered and the corresponding steady-state spinning frequency and whirl orbit amplitude are analytically derived as functions of the drive and the rotor system parameters.

1. Introduction

The modified Maxwell–Bloch equation [1–5]

$$\ddot{q} + \mathrm{i}\eta\Omega\dot{q} + (\delta + \nu)\dot{q} + \mathrm{i}\nu\Omega q + \kappa q = 0, \quad q = x - \mathrm{i}y \tag{1}$$

is used to model a range of rotationally symmetric planar dynamical systems. The variables in Eq. (1), in the context of a discrete model of a symmetric rotor–shaft system shown in Fig. 1, are as follows: q is a complex variable, η is a gyroscopic parameter, Ω is the rotor spinning speed, δ and ν are parameters related to external (or fixed) damping and internal (or rotating) damping, respectively, and κ is a parameter related to potential forces.

In this simple rotor model consisting of a heavy disk mounted symmetrically on a light flexible shaft, which in turn is supported on two identical rigid bearings, it is assumed that the disk and the shaft are fastened to rotate together and the rotor is perfectly balanced. Torsional modes of vibration are not considered in this study. The stiffness of the shaft, the internal or material damping, and the aerial or external damping are referred to the disk center. In Fig. 1, the disk center is shown in a deflected position during whirling and the springs represent the equivalent bending stiffness of the rotor shaft.

The stability domain of the system whose dynamics is described by Eq. (1) is derived in [1] as

$$\delta > 0, \quad \eta\Omega > \frac{\nu\Omega}{\delta + \nu} - \frac{\delta + \nu}{\nu\Omega}\kappa. \tag{2}$$

In available studies in literature, it is assumed that a constant speed motor, which adjusts its output power to maintain the desired speed against any driven load, drives the rotor shaft. If it is assumed that the disk does not rotate about its diameters (owing to the symmetry of the system) then the gyroscopic parameter is neglected and the inequalities in (2) reduce to the well-known stability domain of a symmetric non-gyroscopic rotor system [6,7] given by

$$R_\mathrm{e} > 0, \quad |\Omega| < \omega_\mathrm{n}\left(1 + \frac{R_\mathrm{e}}{R_\mathrm{i}}\right), \tag{3}$$

where $\kappa = K/m = \omega_\mathrm{n}^2$, $\delta = R_\mathrm{e}/m$, $\nu = R_\mathrm{i}/m$, K is the bending stiffness referred to the disk center, m is the mass of the centrally loaded disk, and R_e and R_i are the equivalent external and internal damping referred to the rotor disk center.

These stability conditions can be derived in a variety of ways [8], e.g. Billharz condition [1–3] or Lienard–Chipart criterion [4,5] applied to the complex characteristic polynomial, Routh stability condition applied to real characteristic polynomial, eigenvalue analysis, energy methods [6,7], etc.

This work precisely deals with such cases where the rotor spinning speed is decided by the dynamics of the motor which is influenced by the source loading, i.e. the spinning and whirling of the rotor shaft. In particular, the nature of the steady-state dynamics will be explored.

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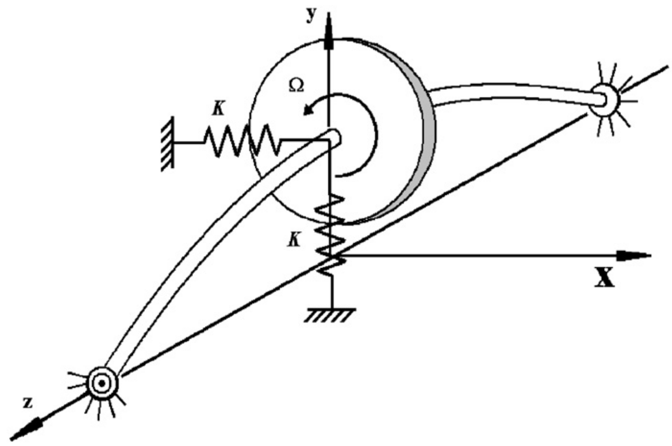

Fig. 1. A Symmetric rotor–shaft system with spinning speed Ω. The stiffnesses of the shaft and the internal and external damping are referred to the rotor disk.

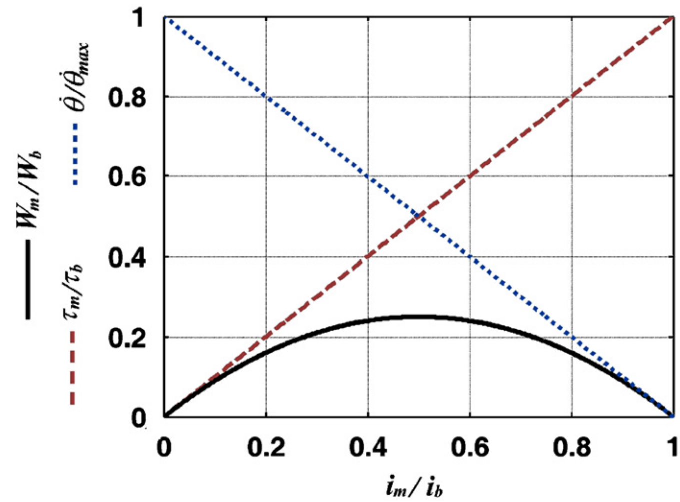

Fig. 2. DC motor characteristics.

2. Source loading of a DC motor drive

In a non-ideal source, the excitation is influenced by the response of the system. The structural response of the system to which an unbalanced non-ideal electrical motor is connected acts like energy sink under certain conditions thereby leading to a kind of jump-phenomena called the Sommerfeld effect [9–15]. This jump-phenomenon is characterized by the inability to realize certain motor speeds near the resonance frequency. Recent research has been mainly focused on control of the passage through resonance with a non-ideal source [16–18]. This work assumes a balanced rotor–shaft system and concerns the influence of a non-ideal DC motor source on the post-critical stability behavior.

Consider that a DC motor drives the rotor shaft. For simplicity, it is assumed that the rotor shaft is torsionally rigid although the obtained results would be still valid without this assumption. The non-linear equations of motion of the system by including the drive dynamics is then given by

$$m\ddot{x} + (R_e + R_i)\dot{x} + R_i\dot{\theta}y + Kx = 0,$$
$$m\ddot{y} + (R_e + R_i)\dot{y} - R_i\dot{\theta}x + Ky = 0,$$
$$I_r\ddot{\theta} + R_r\dot{\theta} = \tau_m - \tau_L, \tag{4}$$

where θ is the rotation of the rotor, R_r is the torsional damping offered to the spinning shaft (due to bearings and the medium), I_r is the rotary inertia of the shaft–disk system about the spinning axis, τ_m is the torque developed by the motor, and τ_L is a torque contributing to the source loading by the non-conservative forces. Note that in the mathematical model of the rotor–motor system given in Eq. (4), the gyroscopic moments acting on the bending modes are neglected (they will be included later). Further note that the non-conservative circulatory forces (terms $R_i\dot{\theta}x$ and $R_i\dot{\theta}y$) appear in Eq. (4) due to rotating material damping of the rotor shaft [6,7,19,20].

A brushed DC motor model is considered as follows:

$$i_m = \frac{V_s - V_e}{R_m} = \frac{V_s - \mu_m\dot{\theta}}{R_m}, \quad \tau_m = \mu_m i_m, \tag{5}$$

where V_s is a constant voltage applied across the motor's terminals, μ_m is a motor characteristic, i_m is the current drawn by the motor, $V_e = \mu_m\dot{\theta}$ is the back EMF developed in the motor coils, R_m is the electrical resistance of the coils, and τ_m is the torque developed by the motor. The mechanical power developed by the motor is $W_m = \tau_m\dot{\theta}$ and the power lost through electrical resistances is $W_l = i_m^2 R_m$.

For $\dot{\theta} = 0$, i.e. the braking condition, the brake torque is $\tau_b = \mu_m V_s/R_m$, the corresponding current drawn is $i_b = V_s/R_m$ and brake power is $W_b = V_s^2/R_m$. The maximum unloaded motor speed is $\dot{\theta}_{max} = V_s/\mu_m$. The characteristics of the considered DC motor are then obtained as shown in Fig. 2, where all plotted variables are non-dimensional quantities.

The system considered in Eq. (4) is a rotationally symmetric (SO2 symmetric) [21] planar dynamical system. Therefore, it is justified to select a rotationally symmetric solution, e.g. point, circle and spiral, for Eq. (4). We seek a steady-state SO2 symmetric solution, which happens to be a circular motion about the equilibrium point. In [22], energy considerations have been used to analyze a continuous Beck's column. It has been argued that harmonic vibrations characterized by a constant energy state exist at the critical load [22–24]. Likewise, properties of the asymptotic state of a tippe top have been analyzed in [4] by finding a constant energy state where the slip velocity vanishes or alternatively by taking orbital derivative of the total energy.

By removing the spatial variable from the harmonic vibrations assumed in [22], the SO2 symmetric steady-state motion of the disk is considered as

$$x = A\cos(\omega t + \psi), \quad y = A\sin(\omega t + \psi), \tag{6}$$

where A is an unknown amplitude, ω is an unknown whirling frequency, and ψ is an arbitrary phase. Moreover, let the steady-state rotor spinning speed be $\dot{\theta}|_{t\to\infty} = \omega_m$. Substitution of Eq. (6) into the first two equations in Eq. (1) and exclusion of the trivial solution $A = 0$ yields

$$\omega = \omega_n = \pm\sqrt{K/m},$$
$$\omega_m = \omega(1 + R_e/R_i). \tag{7}$$

Obviously, if the motor spinning is considered to be positive, one has to deal with only the positive whirl because the backward whirl would not satisfy the second condition in Eq. (7), which agrees with well-known results pertaining to stability of the backward whirl in rotor systems.

The mechanical power drawn from the motor during steady-state whirling is

$$W_m = \tau_m\omega_m = \mu_m i_m\omega_m = \frac{\mu_m(V_s - \mu_m\omega_m)\omega_m}{R_m}$$
$$= \frac{\mu_m}{R_m}\left(V_s - \mu_m\omega\left(1 + \frac{R_e}{R_i}\right)\right)\omega\left(1 + \frac{R_e}{R_i}\right). \tag{8}$$