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Mechanism of buckling development and strain reversal occurrence in elastic–plastic cylindrical shells under axial impact

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ABSTRACT

In order to clarify the mechanism of loading and unloading, buckling growth, and strain-reversal occurrence in elastic–plastic cylindrical shells that are impacted axially, non-linear dynamic equations in incremental form are derived and solved by using the finite difference method. The validity of the developed theory is verified by comparing theoretical and experimental results. From the calculation results, it is found that, at the initial stage of the impact process, the effective stress related to the middle-surface stresses in the part near the impacted end rises with time, and exhibits plastic loading. The radial restraints on the impacted end have obvious influence on the strain reversal occurrence at an early stage of the impact process. The effective modulus controls the post-buckling deformation of the shell, which contrasts with the situation for the bar to be impacted axially.

1. Introduction

The dynamic plastic buckling of cylindrical shells under axial impact has been studied theoretically and experimentally [1–8]. Lepik [5] presented the bifurcation analysis of elastic and plastic cylindrical shells considering the effect of stress wave. Karagiozova and Jones [6,7] studied the axisymmetric buckling of elastic–plastic cylindrical shells under axial impact by using a discrete model, where the stress wave propagation effects were analyzed for moving and stationary specimens. Wang and Tian [8] presented the twin characteristic parameter solution to obtain the critical buckling time, inertial exponential parameter and buckling modes for the axisymmetric dynamic buckling of cylindrical shells under axial elastic–plastic compression waves.

In the theoretical investigation on elastic–plastic buckling of bars and cylindrical shells under axial impact, it was usually supposed that the strain rate associated with the unperturbed motion dominates the strain rate introduced by the perturbed motion so that no strain-rate reversal occurs until the buckling is well developed [2].

In Ref. [9], the strain-rate reversal problem was investigated numerically for a set of bar specimens impacted on a rigid wall. It was found that, in a comparatively long time after impact initiation, $0 \leqslant t \leqslant 10t_{cr}$, where t_{cr} is the critical buckling time, the bar undergoes plastic loading and no strain reversal occurs in the bar.

In this paper, in order to clearly explain the mechanism of loading and unloading, buckling development and strain-reversal occurrence in a thick cylindrical shell under axial high-velocity impact, non-linear dynamic equations in incremental form are derived, and solved by using the finite difference method. For the shell made of a linear strain-hardening material, Mises' criterion is used to derive the constitutive relation between the middle-surface stress increments and middle-surface strain increments. The effective modulus or strain-hardening modulus is used to describe the constitutive relation for bending deformation, depending on whether the strain reversal occurs or not during the impact process. The validity of the presented method is verified by comparing the predicted results with the experimental results in Refs. [1,2]. Moreover, the process of buckling deformation development in the shells is investigated in detail.

2. Dynamic equations for thick cylindrical shells under axial impact

2.1. Two types of axial impact for cylindrical shells

In the present analysis, the dynamic elastic–plastic response of thick cylindrical shells under two types of axial impact is studied. For the first type, as shown in Fig. 1, the shell with an attached mass G_1 is impacted against a rigid wall at the initial velocity v_0. For the second type, as shown in Fig. 2, the stationary shell is impacted by a mass G_2 traveling with an initial velocity v_0. We assume that the shell has a length L, a mean radius R and thickness h, and is

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Nomenclature

L, R, h	initial length, mean radius and thickness of the shell specimen, respectively
ρ, σ_s, E, E_t	density, yield stress, Young's modulus and strain-hardening modulus of the shell material, respectively
r	the geometrical parameter is defined by Eq. (2.24b)
c_0	propagation speed of the axial elastic wave in the shell
G_0	mass of the shell material
G_1	mass attached to the shell impacting against a rigid wall
G_2	mass striking the stationary shell
v_0	initial velocity of the shell impacting against a rigid wall, or initial velocity of the mass striking the stationary shell
x, θ, t	axial, circumferential coordinates, and time variable, respectively
u, w	axial and radial displacement components of the middle surface of the shell
Δt	small increment of the time variable t
ϑ	cross-section rotation of the impacted shell
N_x, N_θ	in-plane force intensities in the impacted shell
M_x	bending moment intensity in the impacted shell
σ_x, σ_θ	axial and circumferential average stresses on the cross-section of the shell, respectively
$\Delta\sigma_x, \Delta\sigma_\theta$	increments of σ_x and σ_θ from the instant t to $t+\Delta t$, respectively
$\Delta\varepsilon_x, \Delta\varepsilon_\theta$	increments of the middle-surface strains
$\Delta\varepsilon_M$	bending strain increment of the outer-surface material on the convex side of the shell
σ_e	effective stress, see Eq. (2.7a)
$d\sigma_e$	effective stress increment, see Eq. (2.9)
$\Delta\varepsilon_x^p, \Delta\varepsilon_\theta^p$	plastic parts of the strain increments $\Delta\varepsilon_x$ and $\Delta\varepsilon_x$ respectively
$d\varepsilon_p$	equivalent plastic strain increment related to $\Delta\varepsilon_x^p$ and $\Delta\varepsilon_\theta^p$ (Eq. (2.7b))
H'	hardening parameter of the linear hardening material
ξ	non-dimensional coordinate parameter related to x (Eq. (2.25a))
τ	non-dimensional time parameter related to t (Eq. (2.25b))
κ	geometrical parameter of the shell (Eq. (2.25c))
$\bar{u}^t, \bar{w}^t$	non-dimensional displacement parameters related to u^t and w^t, respectively (Eqs. (2.26a), (2.26b))
$\bar{\sigma}_x^t, \bar{\sigma}_\theta^t$	non-dimensional stress parameters related to σ_x^t and σ_θ^t, respectively (Eqs. (2.27a), (2.27b))
k_1	friction-factor between the impacted end of the shell and the surface of the rigid wall
k_2	friction-factor between the un-impacted end of the shell and the surface of the attached mass
t_d	impact duration
δ_L	axial shortening of the shell caused by the axial impact

made of the linear strain-hardening material with density ρ, yield stress σ_s, Young's modulus E and strain-hardening modulus E_t.

2.2. Dynamic equations for in-plane forces and bending moment

It is known that a thick cylindrical shell with small radius-to-thickness ratios is deformed in an axisymmetric pattern when it is subjected to axial impact. We use x, θ and t to denote the axial and circumferential coordinates and the time variable, respectively. From the theory of thick shells [10], we derive the following governing equations for the axisymmetric dynamic response of thick cylindrical shells:

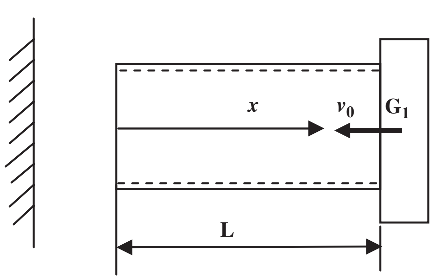

Fig. 1. The traveling cylindrical shell impacted on a rigid wall.

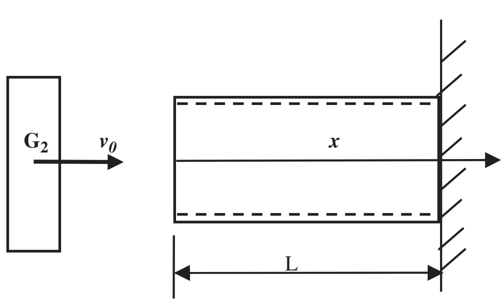

Fig. 2. The stationary cylindrical shell impacted by a mass.

$$N_{x,x} = \rho h\left(u_{,tt} + \frac{1}{12}\frac{h^2}{R}\vartheta_{,tt}\right), \tag{2.1a}$$

$$M_{x,xx} + (N_x w_{,x})_{,x} - \frac{N_\theta}{R} = \rho h\left(w_{,tt} + \frac{1}{12}\frac{h^2}{R}u_{,xtt} + \frac{1}{12}h^2\vartheta_{,xtt}\right), \tag{2.1b}$$

In Eqs. (2.1a), (2.1b), u and w are the displacement components of the middle surface in the axial and radial directions, respectively, ϑ is the cross-section rotation, N_x and N_θ are the in-plane force intensities, M_x is the bending moment intensity, and the subscript x or t preceded by comma denote differentiation with respect to x or t. For simplification, let us estimate the magnitude order of the terms at the right side of Eqs. (2.1a), (2.1b):

$$\vartheta = O(w_{,x}), \tag{2.2a}$$

$$\frac{1}{12}\frac{h^2}{R}\vartheta_{,tt} = O\left(\frac{1}{12}\frac{h^2}{lR}w_{,tt}\right), \tag{2.2b}$$

$$\frac{1}{12}\frac{h^2}{R}u_{,xtt} = O\left(\frac{1}{12}\frac{h^2}{lR}u_{,tt}\right), \tag{2.2c}$$

$$\frac{1}{12}h^2\vartheta_{,xtt} = O\left(\frac{h^2}{12l^2}w_{,tt}\right). \tag{2.2d}$$

In Eqs. (2.2a)–(2.2d), l denotes the variation length of the displacements u and w along the axial direction of the shell, and is half