Available online at www.sciencedirect.com
ScienceDirect
International Journal of Non-Linear Mechanics 43 (2008) 490–496
INTERNATIONAL JOURNAL OF NON-LINEAR MECHANICS
www.elsevier.com/locate/nlm

Solar sail dynamics in the three-body problem: Homoclinic paths of points and orbits

Thomas J. Waters*, Colin R. McInnes

Department of Mechanical Engineeering, University of Strathclyde, Glasgow, UK

Received 22 March 2007; received in revised form 19 November 2007; accepted 4 January 2008

Abstract

In this paper we consider the orbital dynamics of a solar sail in the Earth–Sun circular restricted three-body problem. The equations of motion of the sail are given by a set of non-linear autonomous ordinary differential equations, which are non-conservative due to the non-central nature of the force on the sail. We consider first the equilibria and linearisation of the system, then examine the non-linear system paying particular attention to its periodic solutions and invariant manifolds. Interestingly, we find there are equilibria admitting homoclinic paths where the stable and unstable invariant manifolds are identical. What is more, we find that periodic orbits about these equilibria also admit homoclinic paths; in fact the *entire* unstable invariant manifold winds off the periodic orbit, only to wind back onto it in the future. This unexpected result shows that periodic orbits may inherit the homoclinic nature of the point about which they are described.

Keywords: Solar sail; Homoclinic; Periodic orbit; Three-body problem

1. Introduction

A solar sail is a novel type of spacecraft which uses the radiation pressure of photons reflecting off large sails as its impulse (see [1] for a detailed description). They are therefore of interest as they do not require fuel in the traditional sense. In addition, solar sails are capable of trajectories and orbits which are beyond conventional spacecraft (see for example the GeoSail mission [2]). A natural setting for the solar sail is the circular restricted three-body problem (CR3BP) where the Earth and the Sun are the primary bodies. This is partly because the three-body problem more accurately describes solar system dynamics than the 2-body problem, but also because in the three-body problem there are regions where the gravitational forces on the sail due to the primaries cancel each other, and hence the radiation pressure force on the sail plays a more dominant role. Also, the demands on sail efficiency would be less as the gravitational forces are less, and thus the applications of this analysis are more in the near-term.

A standard procedure in analysing a non-linear system of ODE's is to find its equilibria or fixed points, linearise about these, and use the information from linear order to inform an analysis of the non-linear system; this is the procedure we will follow in this paper. In particular we will direct our attention to the non-linear system's periodic orbits and invariant manifolds. There has been some work carried out on the dynamics of solar sails in the CR3BP. McInnes et al. [3] first described the surfaces of equilibrium points, and some possible uses of same. In Baoyin and McInnes [4] and McInnes [5], the authors describe periodic orbits about equilibrium points in the solar sail three-body problem, however they consider only equilibrium points on the axis joining the primary masses, corresponding to artificial Lagrange points. Such orbits are analogous to the classical 'halo' orbits (where by classical we mean the particle is only acted upon by gavitational forces), which are well documented, for example Farquhar [6], Farquhar and Kamel [7], Breakwell and Brown [8], Richardson [9], Howell [10] and Thurman and Worfolk [11]. With regards to the invariant manifolds, there has been much analysis of the invariant manifolds of halo orbits in the classical problem for the sake of efficient transfer; for example the Genesis mission trajectory was

* Corresponding author.
E-mail address: thomas.waters@strath.ac.uk (T.J. Waters).

0020-7462/$ - see front matter © 2008 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ijnonlinmec.2008.01.001

designed using this technique (see [12]), and a 'petit grand tour' of Jovian moons has been proposed using a similar analysis (see [13]). Some homoclinic paths for the classical triangular points have been found for large mass ratios (see for example [14]), and homoclinic paths can exist for collinear points with particular mass ratios (see [15, Section 9.9.2]). Certain isolated homoclinic paths have been found for periodic orbits about the collinear Lagrange points (see for example [16]), however no periodic orbit whose invariant manifold is made up entirely of homoclinic paths has been found thus far, to the best of our knowledge.

The structure of the paper is as follows: in the next section we will describe the setting of the problem and the equations of motion of the solar sail, as well as the equilibrium points. Section 3 considers the system linearised about equilibrium and the form of the linear solutions. In Section 4 we briefly describe the Lindstedt–Poincaré perturbation method used to find non-linear approximations to periodic orbits, and in Section 5 we examine the invariant manifolds of equilibria and periodic orbits. We find a large variety in the position, inclination, amplitude and frequency of periodic solutions to the equations off motion, and unexpected homoclinic paths associated with equilibria and periodic orbits which have no analogue in the classical problem. Such results suggest that the solar sail CR3BP presents a rich and complex model, the intricate details of which are only beginning to become apparent.

2. Equations of motion in the rotating frame

We follow the conventions set out in McInnes [1]. We consider a rotating coordinate system in which the primary masses are fixed on the x-axis with the origin at the centre of mass, the z-axis is the axis of rotation and the y-axis completes the triad. We chose our units to set the gravitational constant, the sum of the primary masses, the distance between the primaries, and the magnitude of the angular velocity of the rotating frame to be unity. We shall denote by $\mu = 3 \times 10^{-6}$ the dimensionless mass of the smaller body m_2, the Earth, and therefore the mass of the larger body m_1, the Sun, is given by $1 - \mu$ (see Fig. 1).

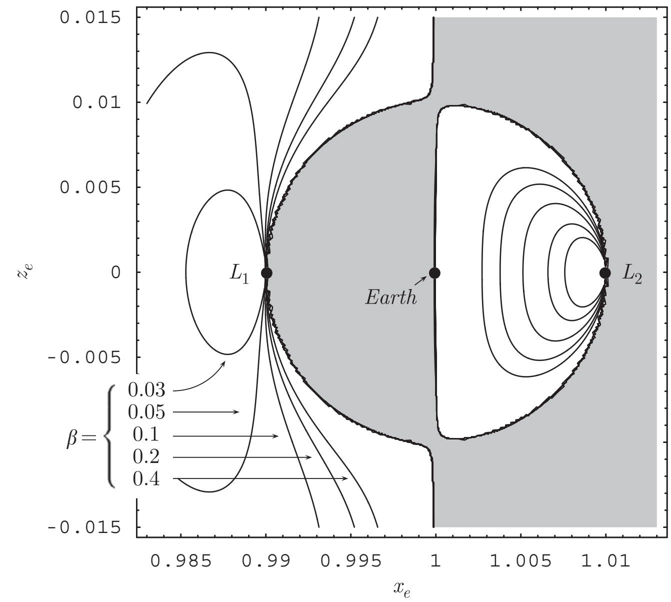

Fig. 2. Surfaces of equilibrium points in the x_e–z_e parameter space. Each curve is specified by a constant value of β, and the position of the equilibrium point along the curve is given by γ. The grey shaded regions denote areas where equilibrium is not possible.

Denoting by $\boldsymbol{r}, \boldsymbol{r}_1$ and $\boldsymbol{r}_2$ the position of the sail w.r.t. the origin, m_1 and m_2, respectively, the solar sail's equations of motion in the rotating frame are

$$\frac{\mathrm{d}^2\boldsymbol{r}}{\mathrm{d}t^2} + 2\boldsymbol{\omega} \times \frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}t} = \boldsymbol{a} - \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{r}) - \nabla V \equiv \boldsymbol{F}, \tag{1}$$

with $\boldsymbol{\omega} = \widehat{\boldsymbol{z}}$ and $V = -[(1-\mu)/r_1 + \mu/r_2]$ where $r_i = |\boldsymbol{r}_i|$. These differ from the classical equations of motion in the CR3BP by the radiation pressure acceleration term

$$\boldsymbol{a} = \beta \frac{(1-\mu)}{r_1^2} (\widehat{\boldsymbol{r}}_1 \cdot \boldsymbol{n})^2 \boldsymbol{n}, \tag{2}$$

where β is the sail lightness number, and is the ratio of the solar radiation pressure acceleration to the solar gravitational acceleration. Here $\boldsymbol{n}$ is the unit normal of the sail and describes the sail's orientation. We define $\boldsymbol{n}$ in terms of two angles γ and

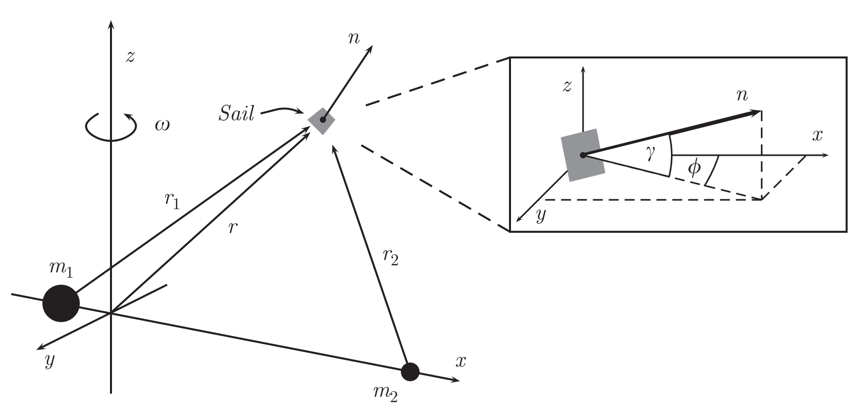

Fig. 1. The rotating coordinate frame and the sail position therein. The angles γ and ϕ which the sail normal makes with respect to the rotating frame are also shown.