Cement and Concrete Composites 91 (2018) 108–117

Contents lists available at ScienceDirect

Cement and Concrete Composites

journal homepage: www.elsevier.com/locate/cemconcomp

Probabilistic-based assessment for tensile strain-hardening potential of fiber-reinforced cementitious composites

Junxia Li[a,b], En-Hua Yang[c,*]

[a] Interdisciplinary Graduate School, Nanyang Technological University, 50 Nanyang Avenue, 639798 Singapore
[b] Residues & Resource Reclamation Centre, Nanyang Environment and Water Research Institute, Nanyang Technological University, 1 Cleantech Loop, 637141 Singapore
[c] School of Civil and Environmental Engineering, Nanyang Technological University, 50 Nanyang Avenue, 639798 Singapore

ARTICLE INFO

Keywords:
Tensile strain-hardening
Fiber-reinforced cementitious composites (FRCC)
Engineered cementitious composites (ECC)
Strain-hardening cementitious composites (SHCC)
Multivariate adaptive regression splines (MARS)
First order reliability method (FROM)

ABSTRACT

This paper presents a novel probabilistic-based approach considering material heterogeneity to assess the tensile strain-hardening potential of fiber-reinforced cementitious composites (FRCC). Multivariate adaptive regression splines (MARS) method is used to explicitly express the performance indices governing tensile strain-hardening. First order reliability method (FROM) is then carried out to evaluate tensile strain-hardening potential of FRCC. Results show that strain capacity of FRCC has a negative correlation with failure probability and it increases exponentially with decreasing failure probability. Analysis of variance (ANOVA) decomposition of MARS model indicates increasing fiber strength and volume, reducing fiber modulus, and moderate interface frictional bond are effective means to improve tensile strain-hardening potential of FRCC. The proposed approach is thus able to consider uncertainty in evaluating tensile strain-hardening potential of FRCC by treating micromechanical parameters as random variables and taking heterogeneity into account in the probabilistic-based model.

1. Introduction

Fibers have been used to address the brittleness of cement-based material. Through proper tailoring, studies have shown fiber-reinforced cementitious composites (FRCC) can even exhibit tensile strain-hardening behavior with high tensile ductility of several percent, e.g. engineered cementitious composites (ECC) [1,2] or strain-hardening cementitious composites (SHCC) [3,4]. Tensile strain-hardening of FRCC is fundamentally governed by two criteria developed by Li and co-workers [5] as shown below.

$$J_b^{'} = \sigma_0\delta_0 - \int_0^{\delta_0} \sigma(\delta)d\delta \geq J_{tip} \quad (1)$$

$$\sigma_0 \geq \sigma_c \quad (2)$$

where $J_b^{'}$ is the complementary energy which can be calculated from the bridging stress σ versus crack opening δ curve, σ_0 is the maximum bridging stress corresponding to the opening δ_0, $J_{tip}=K_m{}^2/E_m$ is the crack tip toughness where K_m is the matrix fracture toughness and E_m is the matrix Young's modulus, and σ_c is the matrix tensile cracking strength governed by the matrix fracture K_m and flaw size a_0. The bridging stress-crack opening relation $\sigma(\delta)$ can be determined with an analytical model, involving a set of ten micromechanical parameters describing the fiber and the fiber/matrix interface properties. A numerical solution for the analytical model was proposed by Yang et al. [27] to calculate the $\sigma(\delta)$ as well as $J_b^{'}$.

Eqn. (1) ensures that the steady state crack propagation prevails under tension, while Eqn. (2) is the criterion to allow development of multiple cracks. Satisfactory of both equations is necessary to achieve the tensile strain-hardening of FRCC. This model has been used to guide ingredients selection and component tailoring to achieve tensile strain-hardening performance at minimum fiber content [2,5]. For instance, tailoring of fiber types and geometry [6], tailoring of fiber/matrix interface through fiber surface modification [7,8], and control of matrix flaw size and distribution [9] were guided through the tensile strain-hardening assessment of the composite. It is taken as a deterministic model assuming uniform fiber, matrix, and interface properties and thus constant micromechanical parameters are used as inputs to evaluate the tensile strain-hardening potential. In reality; however, heterogeneous nature of cement-based material, FRCC in particular, is inevitable. Heterogeneity may originate from variation of properties of ingredients such as inconsistent fiber diameter and fiber strength, processing (variation of flaw size and distribution and non-uniform fiber distribution [10]), and curing.

To ensure robust tensile strain-hardening taking into account of material variability, one approach is to propose large margins of $J_b^{'}/J_{tip}$

* Corresponding author.
E-mail address: ehyang@ntu.edu.sg (E.-H. Yang).

https://doi.org/10.1016/j.cemconcomp.2018.05.003
Received 17 July 2017; Received in revised form 13 March 2018; Accepted 1 May 2018
Available online 08 May 2018

and σ_0/σ_c [11,12]. It is analogy to the build-in safety factor of the prescriptive-based design of structures. It has been demonstrated experimentally that ECC reinforced with polyethylene (PE) fibers requires $J_b^{'}/J_{tip} > 3$ and $\sigma_0/\sigma_c > 1.2$ to ensure saturated multiple cracking and robust tensile strain-hardening [13], while polyvinyl alcohol (PVA) fiber-reinforced ECC requires an even higher σ_0/σ_c of 1.45 and above [14]. This empirical-based approach requires large amount of testing and the values are only limited to the specific SHCC system of interest. Any changes to the ingredients and proportions of the SHCC may require re-establishment of such safety factor. Thus, there is a need to develop a holistic approach which can be applied universally and adopted for tailoring different SHCC system.

This paper presents a probabilistic approach to achieve this goal. In the approach, heterogeneity is considered in the micromechanical model to capture the uncertainty and to evaluate the tensile strain-hardening potential of FRCC. It is realized by taking all micromechanical parameters, including fiber properties (i.e. fiber volume V_f, fiber modulus E_f, fiber length L_f, fiber diameter d_f, and fiber tensile strength σ_{fu}), interface properties (i.e. chemical bond strength G_d, frictional bond strength τ_0, slip hardening coefficient β, fiber strength reduction coefficient f', snubbing coefficient f), and matrix properties (i.e. matrix modulus E_m, matrix fracture toughness K_m, flaw size c), as random variables to capture sources of randomness. It is noteworthy that the randomness of fiber orientation and fiber embedment length have been accounted for in the micromechanical model $\sigma(\delta)$ by adopting probability density functions that describe the spatial variability of the fibers [16] in 2D. Other randomness, such as variation of raw ingredients, processing induced variation of flaw size and distribution and curing induced variation of matrix and interface properties, can be captured by adopting suitable probability density functions that describe the variability. However, randomness of fiber distribution caused by the formwork geometry and mixing induced uncertainty of quality of fiber dispersion are not considered in the current approach. As such, fiber bridging $\sigma(\delta)$ as well as $J_b^{'}$, J_{tip}, σ_0, and σ_c would also become random variables, and thus the reliability of the two tensile strain-hardening criteria (Eqns. (1) and (2)), i.e. tensile strain-hardening potential of FRCC, can be quantitatively assessed. To implement this probabilistic-based assessment, multivariate adaptive regression splines (MARS) models are firstly created to obtain an explicit mathematical expression of $J_b^{'}$ and σ_0 instead of integral analytical form [15] and numerical form [16]. Probabilistic assessment is then conducted on FRCC mixes by means of first order reliability method (FROM) to assess the reliability.

2. Conceptual framework of the proposed method

Satisfactory of both Eqns. (1) and (2) is necessary for tensile strain-hardening and therefore the boundary separating the safe and failure domain is the limit state surface (performance function [17]) which can be defined as

$$G(x) = J_b^{'} - J_{tip} = 0 \tag{3}$$

$$F(x) = \sigma_0 - \sigma_c = 0 \tag{4}$$

where x denotes all micromechanical parameters as random variables. Mathematically, $G(x) > 0$ and $F(x) > 0$ denote the 'safe' domain, while $G(x) < 0$ and $F(x) < 0$ denote the 'failure' domain. In this case, the safe domain represents the possession of tensile strain-hardening behavior of FRCC, and the failure domain stands for tensile strain-softening behavior. The limit state surface $G(x)$ and $F(x)$ are required to be known explicitly. Thus, closed-form mathematical functions of $J_b^{'}$ and σ_0 are constructed through MARS, which is a nonparametric regression method to model the nonlinear responses between input variables and output introduced by Friedman in 1991 [18]. The main advantages of MARS are of its capacity to deal with high dimensional data and easy to interpret the model. Extensive applications of MARS include estimating the deformation of asphalt mixtures, analyzing shaking table tests of reinforced soil wall and analysis of geotechnical engineering systems [19–21]. J_{tip} is already an immediate function of K_m and E_m. As for σ_c, it is simply approximated to be the matrix cracking strength σ_{mu} in the current study.

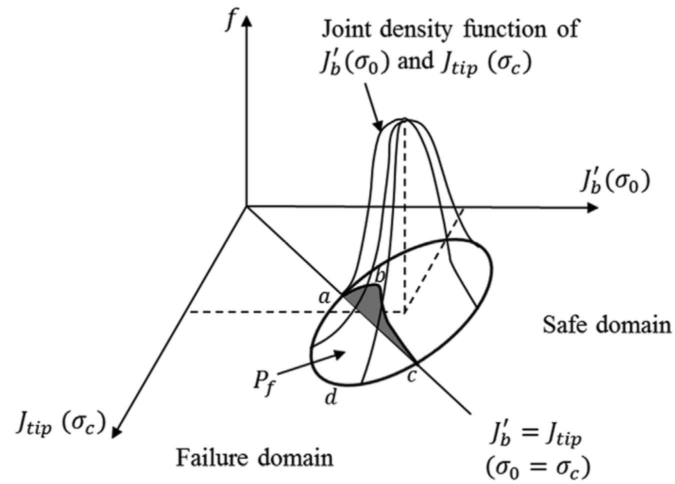

Fig. 1. 3D joint PDF.

Reliability is defined as the probability of the performance function $G(x)$ or $F(x)$ greater than zero, i.e. $P\{G(x) > 0\}$ and $P\{F(x) > 0\}$. The tensile strain-hardening potential of FRCC is evaluated with the concept of failure probability P_f. The calculation of the probability of failure P_f is the integration of the probability density function (PDF) over the failure domain as indicated by the volume *abcd* in Fig. 1. Many methods can be used for probabilistic assessment, such as response surface method (RSM) and Monte Carlo Simulation (MCS). FORM is used in the current study, where random variables are assumed as standard normal distributions, and thus P_f can be calculated by

$$P_{f1} = P\{G(x) < 0\} \approx \phi(-\beta_1) \tag{5}$$

$$P_{f2} = P\{F(x) < 0\} \approx \phi(-\beta_2) \tag{6}$$

where P_f is the probability that the failure event occurs; ϕ is the distribution function of the standard normal distribution; and β is the reliability index computed as

$$\beta = min\sqrt{\left(\frac{x_i-\mu_i}{\sigma_i}\right)^T [R]^{-1}\left(\frac{x_i-\mu_i}{\sigma_i}\right)} \tag{7}$$

where x_i is the random variable; μ_i and σ_i are the mean value and the standard deviation of the random variable, respectively; and R is the correlation matrix.

3. Mathematical expression of $J_b^{'}$ and σ_0 through MARS

3.1. Database for $J_b^{'}$ and σ_0

In seek of MARS models for $J_b^{'}$ and σ_0, a sound and large enough database is created using the numerical solution developed by Yang et al. [16]. A number of groups of inputs and outputs are acquired for establishment of the prediction model by regression fitting method. In this study, ten relevant micromechanical parameters (denoting $x1$ to $x10$ as input variables) and two performance index (denoting $y1$ and $y2$ as outputs) are shown in Table 1, the selected values are typical ones in PE fiber [8], PVA fiber [22] and PP fiber reinforced cementitious composites [6]. All possible combinations of input variables are used to get corresponding outputs, except that some outliers are removed using the residual analysis because it may lead to crude model. Thereof, 527 groups (75%) of the observations were randomly selected for training