International Journal of Pressure Vessels and Piping 135-136 (2015) 26–35

Contents lists available at ScienceDirect

International Journal of Pressure Vessels and Piping

journal homepage: www.elsevier.com/locate/ijpvp

A comparison of various plastic work curvature methods

C.K. Seal [a, *], J.-J. Han [b], R.A. Ainsworth [a]

[a] The University of Manchester, Oxford Road, Manchester M13 9PL, United Kingdom
[b] Korea University, Anam-Dong, Seongbuk-Gu, Seoul 136-701, Republic of Korea

ARTICLE INFO

Article history:
Received 16 September 2014
Received in revised form
5 October 2015
Accepted 14 October 2015
Available online 19 October 2015

Keywords:
Plastic collapse
Plastic work curvature
FEM

ABSTRACT

The use of plastic work as a means of determining the plastic collapse load of a structure is a promising new approach. Several methods of assessing the plastic limit have been proposed in recent publications but there is, as yet, no clear consensus as to the best approach to take. This article compares the various methods proposed and recommends a new approach which is not subjective and which is considered to be more reliable than the twice elastic slope load.

The recommended method plots normalised work versus normalised load, and uses a curve fitting method to estimate the plastic load. It is illustrated by application to several pressure vessels.

1. Introduction

The plastic work curvature method is a promising new approach to determining the plastic limit of a structure, especially one that is experiencing a complex combination of loads. The difficulty in applying this method, however, is in determining what level of curvature corresponds to the onset of plastic collapse, and thus the plastic limit. During the development of the method, various plastic limits have been proposed. This work seeks to assess these different methods and to compare the resulting plastic loads to the established twice elastic slope method.

The simplest elastic–plastic analysis that is used to determine the plastic collapse load is limit analysis. Limit analysis uses an elastic-perfectly plastic material model and small deformation theory to determine the collapse load. In this case, the ability of the structure to carry load is limited by the redistribution of stress within the structure and the limit load is identified when a plot of load versus deformation reaches a plateau, as illustrated in Fig. 1. At this point, the structure cannot accommodate any further increase in load by stress redistribution and the structure fails due to unconstrained plastic deformation. In practice, however, the ASME twice elastic slope (TES) method and the tangent intercept (TI) method are more commonly used to determine the design collapse load [1].

The TES load is determined by plotting an appropriate load versus deformation curve. From this curve, the elastic slope can be determined and the TES line constructed as shown in Fig. 2. The angle that the TES line makes with the ordinate axis is determined from the angle between the elastic line and the ordinate axis from:

$$\tan\phi = 2 \times \tan\theta \tag{1}$$

where ϕ is the angle between the ordinate axis and the TES line, and θ is the angle between the elastic line and the ordinate axis.

The TI load is similarly determined from an appropriate load versus deformation curve. In this case, a line tangential to the plastic component of the curve is constructed and the elastic line extended to intercept this tangent. The load that corresponds to this intercept is the TI load. This procedure is illustrated in Fig. 3.

Both of these methods have limitations that have been explored by various authors [2–6]. In both cases the selection of an appropriate load and deformation can be difficult, especially where there are combined loadings.

The TES line may not intersect the load versus deformation curve, as discussed by Robertson et al. [4]. Kirkwood and Moffat [2] and Moffat et al. [3] showed that the elastic behaviour of the structure, remote from the location where plastic collapse occurs, affects the solution, yielding different TES loads for nominally the same collapse mechanism.

The TI method relies on the construction of an appropriate tangent to the plastic component of the load versus deformation curve and it is not always clear from the curve where this tangent should be drawn [6].

* Corresponding author.
E-mail addresses: christopher.seal@manchester.ac.uk (C.K. Seal), intelligentj@korea.ac.kr (J.-J. Han), Robert.Ainsworth@manchester.ac.uk (R.A. Ainsworth).

http://dx.doi.org/10.1016/j.ijpvp.2015.10.001
0308-0161/

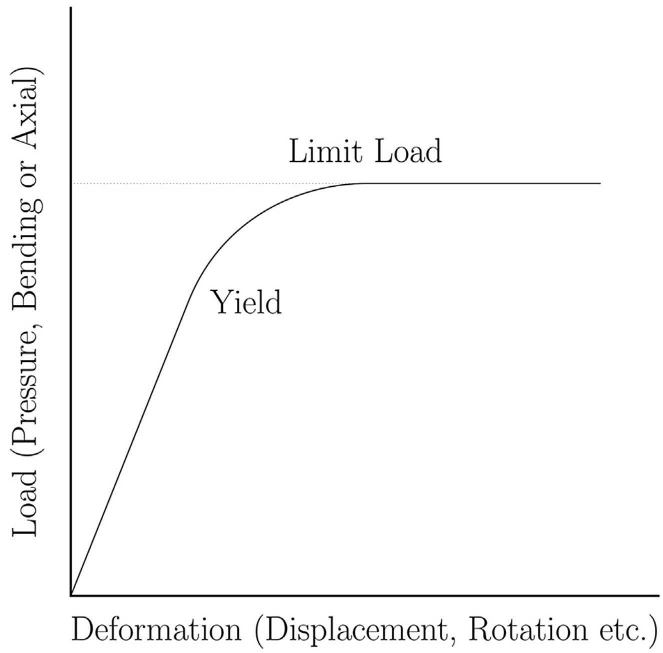

Fig. 1. Schematic of limit load assessment.

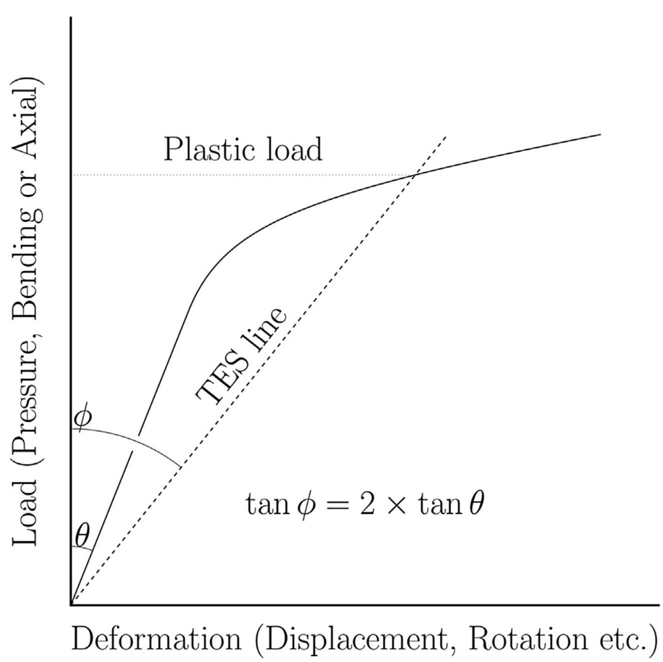

Fig. 2. Construction of the twice elastic slope line.

These limitations have driven recent work [6–12] that seeks to develop a more robust, physically representative method for estimating the plastic collapse load. This paper takes these developments further. Section 2 first briefly summarises proposed plastic work methods. Section 3 then sets out plastic work procedures for estimating the plastic collapse load and addresses how the curvature required in some of these methods may be defined. Section 4 then describes the simple and more complex pressure vessel geometries under single and combined loadings to which the methods of Section 3 are applied. Section 5 then presents and

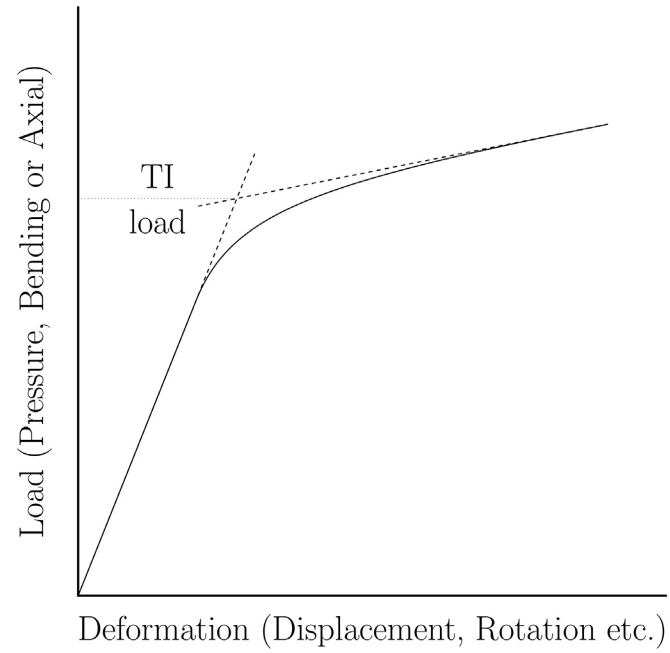

Fig. 3. Schematic of the tangent intercept method.

discusses the results. Conclusions are presented in Section 6.

2. Proposed plastic work methods

Plastic work can be used as a measure of the collapse load due to the relationship between work and load acting as an indication of how the volume of the structure which is plastically deforming changes. Equation (2) illustrates the relationship between work, load (as indicated by the stress) and the volume of the plastically deforming material.

$$W_P = \int_0^{V_P} \int_0^{\varepsilon_P} \sigma(\varepsilon) \cdot d\varepsilon \cdot dV \tag{2}$$

where W_P is the plastic work, V_P is the volume of the material that has undergone plastic deformation and σ and ε are stress and plastic strain respectively.

The idea of using plastic work as a means of assessing plastic collapse was proposed by Gerdeen [13] who suggested that gross plastic deformation be judged to have occurred once the plastic work becomes excessive. Muscat et al. [7], developed this idea into a criterion for determining the plastic collapse limit from a plot of load versus plastic work.

The approach taken by Muscat et al. was to take a tangent to the load vs plastic work curve at the point at which the curve flattened, and to extend this tangent to the load axis as shown in Fig. 4. The point of intersection between the tangent and the load axis defines the plastic load.

Using plastic work in this way eliminates the need to select an appropriate load and deformation parameter and is thus readily implemented for combined loadings and complex geometries. However, the method proposed by Muscat et al. suffers from the same disadvantage that the TI method does, i.e. it is not always clear where to construct the tangent to the work versus load curve [6,10].