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Analytical prediction and experimental validation for longitudinal control of cable oscillations

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Abstract

The paper summarizes the knowledge acquired from the analytical studies and the experimental implementation of a longitudinal non-collocated control strategy for the reduction of cable oscillations. The control is introduced by imposing a longitudinal action at one support based on the knowledge of transverse displacements and velocities of a few selected points. A spatially one-dimensional continuous model of a suspended cable has been used to describe the main features of the non-collocated longitudinal active control strategy. A discrete modal representation has permitted the introduction of suitable non-linear state-feedback controllers. The results have been used to derive an implementable strategy, based on direct output feedback, which preserves the main previous control features. A physical model of an actively controlled cable has been used to demonstrate the control effectiveness of the proposed strategy through a large campaign of experiments, conducted in various frequency ranges and amplitude levels including meaningful external resonance conditions. The responses predicted by the analytical model and the experimental results show good qualitative agreement with one another, in both the uncontrolled and controlled experienced cable dynamics.

Keywords: Active control; Non-linear dynamics; Cables; Experimental dynamics

1. Introduction

The active control of cable oscillations presents a challenging research problem due to its twofold technical and theoretical nature involving control and non-linear dynamics. The problem is a worthwhile challenge because practically implementable control techniques to suppress cable vibrations will enhance cable structural efficiency, preventing degradation and fatigue damage, and will increase serviceability performance. Yet, the variety of non-linear phenomena that cables present, along with the complexity of their mathematical models [1], introduces appreciable novelty to the control problem [2].

Research on cable dynamics is primarily motivated by recurrent vibration phenomena experienced in different engineering applications involving these important structural elements. Different methods have been proposed to reduce such vibrations. Passive dampers acting in the transverse direction are devices most commonly utilized and they have recently been enhanced through the use of semi-active technology [3]. However, the location of these dampers, generally close to the cable support, limits the maximum achievable modal damping, especially in long cables [4]. A recent alternative approach is the use of an active control scheme utilizing either a transverse [5,6] or a longitudinal [7,8] boundary motion. In the first case, a collocated control configuration is proposed with an imposed out-of-plane movement of the support based on position, velocity and angle measure [5]. The stability properties of the controlled system furnish the basis for an adaptive strategy to control out-of-plane cable oscillations [6]. In the second case, a longitudinal action at the support is used to reduce transverse vibrations, taking advantage of the fact that longitudinal motion is coupled with transverse motion in the range of small amplitude vibrations, as well. Both non-collocated and collocated control configurations have been considered. In the case of the non-collocated control configuration, the longitudinal action relies on the measured transverse displacements and velocities of some monitored points; in the collocated configuration,

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it relies only on physical quantities in the longitudinal direction at the support. Different effects evidenced due to the longitudinal action [7,8] are known as active stiffness control, when associated with cable stretching, and active sag induced force control, when associated with the initial curvature of the cable. The aim of the collocated configuration is to prevent spillover instability, but it has primarily been used to reduce in-plane oscillations [9]. Non-collocated control strategies have also been considered in slack [10] and shallow [11] cables. A previous study by one of the authors has shown that the longitudinal active control scheme requires a careful analysis because geometric non-linearities coupled with control feedback may introduce non-dissipative terms, producing some undesirable effects [10]. Thus, the state-feedback in a polynomial form was analyzed looking separately at the effects of the different terms. In particular, it has been shown that linear and quadratic velocity terms can be effective in reducing in-plane oscillations, while quadratic terms have to be used for the out-of-plane components. Another possibility is a feedback bilinearization of the system, which offers advantages and disadvantages, as shown in [12]. In the cited work, local and global analysis of the controllability of the non-linear discrete system of the cable have been developed. Hence, a state-feedback cancellation of the non-linear terms describing the cable elongation is presented, showing the possibility of obtaining a simpler controlled dynamical system represented in a bilinear form, which assures global asymptotic stabilization of free oscillations in the cable. Finally, in order to evaluate control spillover effects inherent to the proposed non-collocated scheme an enlarged 8-*dofs* model is considered. This model numerically evidences the absence of instabilities in the experienced control region.

The present paper demonstrates of the ability of multimodal longitudinal active control to reduce non-linear cable oscillations. Results previously obtained both theoretically and numerically [8,10,12] are here extended by deeply considering the contribution of the anti-symmetric modes to the cable dynamics, which contribution is strongly evidenced in the novel experimental validation of the studied control strategies. In this respect, an implementable non-collocated control law based on direct-output feedback is presented showing that this approach preserves all the previous features of the developed control algorithms. Then, different refined reduced discrete models, with an increasing number of modes accounted in the description of the relevant cable dynamics, has been used to evaluate numerically the performance of the selected control laws. These analyses have been driven by the dynamics experienced in the laboratory tests. Indeed, despite the nominal absence of internal resonance for the selected cable, the experienced dynamics have shown, under in-plane harmonically forced oscillations, a rich and evident modal coupling between the first two in-plane and two out-of-plane modes, respectively. The main modal interaction phenomena have been described constructing the frequency response curves, namely *frcs* to in-plane symmetric harmonic load, for discrete cable models containing an increasing number of modal components. The numerical findings have been confirmed by experimental results showing that in a wide experienced frequency range of the symmetric harmonic force, selected around the first symmetric modes, a rich modal interaction involving mainly the first four modes can be found even in the absence of internal resonance conditions. These modal components are opportunely captured in the experiments by means of two biaxial follower cameras positioned in two points at $\ell/4$ and $3/4\ell$. The control strategy has been based on a single longitudinal action driven by the direct-output non-linear feedback. On the basis of the measured four transverse components, the strategy has shown control capability in the entire experienced frequency range, overcoming all the encountered motions from the classical symmetric in-plane type to more complicated spatial motion with both symmetric and anti-symmetric components.

2. Analytical model and control strategy

The static equilibrium configuration of a cable suspended between two fixed horizontal supports can be described by a curve that lies in the vertical plane (Oxy in Fig. 1). By referring to taut and shallow cables (i.e. $d/\ell \leqslant \frac{1}{8}$, where d is the cable sag and ℓ is the distance between the two supports), the configuration under self-weight can be described by a parabolic function, $y(x) = 4d[x/\ell - (x/\ell)^2]$, with constant horizontal tension H, assumed to describe the initial tension N_0, $(N_0(s) \simeq H)$.

The evaluation of the static reference configuration C^0 described by the parabolic function $y(x)$, permits the derivation of the dynamic varied configuration C^1 through the displacement components $u(x,t)$, $v(x,t)$ and $w(x,t)$ along the co-ordinate axes x, y and z, respectively.

Following [13], the Lagrangian measure of strain is assumed

$$e(x,t) = u' + y'v' + \tfrac{1}{2}(v'^2 + w'^2) \tag{1}$$

the equations of motion of the system are obtained through the extended Hamilton's principle,

$$\begin{aligned}
&m\ddot{u} + \mu_u \dot{u} - [EAe]' = 0,\\
&m\ddot{v} + \mu_v \dot{v} - [Hv' + EA(y' + v')e]' = 0,\\
&m\ddot{w} + \mu_w \dot{w} - [Hw' + EAw'e]' = 0,
\end{aligned} \tag{2}$$

where E is the modulus of elasticity, A is the area of the cross section, m, μ_u, μ_v and μ_w are the mass and damping coefficients of the cable for unit length; a dot and a prime indicate derivatives with respect to time t and the abscissa x, respectively. The assumption has been introduced that the gradient of the horizontal component of the dynamic displacement is smaller than the gradient of the transversal components ($u' \ll v'$, w'), and $y' \ll 1$, $H/EA \ll 1$, and the problem is completed by boundary conditions in 0, ℓ which take into account both the control action u_c and the vertical synchronous support motion v_g producing the in-plane excitation. In particular, a longitudinal displacement $u_c(t)$ of one support $(x = \ell)$ is imposed as a control action, assuming that $u_c/\ell \ll 1$for all t. With this type of control and external excitation the differential equation of motion (2) is not changed, while the following boundary conditions hold

$$u(0,t) = 0, \quad u(l,t) = u_c(t)$$