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Stagnation-point flow of upper-convected Maxwell fluids

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Abstract

Two-dimensional stagnation-point flow of viscoelastic fluids is studied theoretically assuming that the fluid obeys the upper-convected Maxwell (UCM) model. Boundary-layer theory is used to simplify the equations of motion which are further reduced to a single non-linear third-order ODE using the concept of stream function coupled with the technique of the similarity solution. The equation so obtained was solved using Chebyshev pseudo-spectral collocation-point method. Based on the results obtained in the present work, it is concluded that the well-established but controversial prediction that in stagnation-point flows of viscoelastic fluids the velocity inside the boundary layer may exceed that outside the layer may just be an artifact of the rheological model used in previous studies (namely, the second-grade model). No such peculiarity is predicted to exist for the Maxwell model. For a UCM fluid, a thickening of the boundary layer and a drop in wall skin friction coefficient is predicted to occur the higher the elasticity number. These predictions are in direct contradiction with those reported in the literature for a second-grade fluid.

Keywords: Stagnation-point flow; Second-grade fluid; Maxwell fluid; Boundary layer

1. Introduction

Boundary-layer theory has been the working horse of modern fluid mechanics since its introduction to the engineering world by Prandtl in the early 1990s [1]. That is, over the past century, many engineering fluid mechanical problems have been solved using this theory rendering results which compare well with experimental observations—at least as far as Newtonian fluids are concerned. In spite of the success of this theory for Newtonian fluids, an extension of the theory to non-Newtonian fluids has turned out to be a rather formidable task [2–4]. The main difficulty in reaching to a general boundary-layer theory for non-Newtonian fluids lies obviously in the diversity of these fluids in their constitutive behavior. Further difficulty arises from their simultaneous viscous and elastic properties such that differentiating between those effects which arise as a result of a fluid's shear-dependent viscosity from those which are attributable to the fluid's elasticity becomes virtually impossible. Therefore, it is not surprising that most studies dealing with non-Newtonian boundary layers are concerned mainly with simple rheological models such that these two effects can be addressed separately.

In spite of the incompleteness of current boundary-layer theories for viscoelastic fluids, preliminary studies made on the basis of simple rheological models (albeit admittedly limited in their scope and range of applicability) have served to show that much richer behavior can be expected for non-Newtonian fluids as compared with Newtonian ones [3]. For example, while blowing through an infinite porous fluid has no exact solution for Newtonian fluids, for certain non-Newtonian fluids, on the other hand, an exact solution has been found in this particular flow [5]. Also, it has been shown that while for non-Newtonian fluids boundary layers are formed only at large Reynolds numbers, for non-linear fluids they can form even at small Reynolds numbers [3,4]. But perhaps the most striking characteristic of viscoelastic boundary layers is the notion that boundary layers of different natures (inertial vs. elastic) may develop [3] in these fluids sometimes exhibiting complicated multiple deck structures with different effects dominating in different decks [6].

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The state of despair as to the incompleteness of current boundary-layer theories for viscoelastic fluids can best be seen when dealing with the stagnation-point flow of such fluids. That is, whereas for Newtonian fluids this particular flow renders itself to an exact solution valid at any Reynolds number [7], in contrast, even for one of the simplest viscoelastic fluid models available, i.e., the so-called second-grade model [8], no such an exact solution does exist. As a matter of fact, Rajeswari and Rathna [9] had to rely on boundary-layer approximations in order to obtain an estimation of the wall shear stress in stagnation-point flow of a second-grade fluid. To solve the governing equation, Rajeswari and Rathna [9] made use of the Karman–Pohlhausen momentum integral method [7] and concluded that the wall shear stress becomes larger the higher the fluid's elasticity. Their finding was later corroborated by Davies [10] using a similar approach. In another fundamental work dealing with stagnation-point flows of second-grade fluids, Beard and Walters [11] reduced the governing PDEs to a single non-linear fourth-order ODE using boundary-layer theory combined with the concept of similarity solution. The equation so obtained turned out to be still a difficult problem to solve due to the lack of sufficient physical boundary conditions. Relying on perturbation technique [12] to circumvent the problem with the extra boundary condition, Beard and Walters [11] converted their singular perturbation problem into a regular one by reducing the equations to a system of two third-order ordinary differential equations (good enough to be solved numerically with available boundary conditions). From their numerical results, Beard and Walters [11] reached to the general conclusion that the main effect of a fluid's elasticity is to increase the wall shear stress, as previously reported by Rajeswari and Rathna [9]. But unlike Rajeswari and Rathna [9], and also Davies [10], Beard and Walters [11] predicted an overshoot in the velocity inside the boundary layer as a result of the fluid's elasticity. No experimental data has ever been presented to validate the significance of any of these theoretical predictions.

Beard and Walters' prediction that in stagnation-point flows of viscoelastic fluids velocity inside the boundary layer may exceed that outside the layer has stimulated many researchers over the past 40 years to investigate its validity. Frater [13] appears to be the first to suggest that the overshoot of the velocity in the boundary layer might be due to seeking a regular perturbation solution of the problem in terms of an elasticity number. Recent findings by Teipel [14], Garg and Rajagopal [15], and Pakdemirli and Suhubi [16] have demonstrated that the perturbation technique may not render satisfactory results when dealing with viscoelastic fluids. The shortcomings of the perturbation method in dealing with stagnation-point flows of second-grade fluids has beautifully been demonstrated by Ariel [17]. Using an accurate hybrid method (combining the features of the finite difference technique and the shooting method), Ariel [17] attempted to solve the original fourth-order ODE instead of the perturbed set. He had to augment the number of required boundary conditions to four in order to solve the fourth-order ODE, and this was done by imposing an extra condition at the wall based on the governing equation itself. The results obtained by Ariel [17] turned out to be quite different from those reported by Beard and Walters [11] especially for $k > 0.1$. In another interesting work tackling with the original fourth-order ODE instead of the perturbed set, Serth [18] showed that the wall shear stress calculated using the orthogonal collocation-point method are quite different from those obtained from the perturbation method (with the difference becoming larger the higher the elasticity number k). The work carried out by Garg and Rajagopal in this area [15] is quite striking in that they have shown that the sign adopted by Beard and Walters (and many others) for the elasticity number, k, must be reversed in order for their second-grade model to comply with theromodynamical constraints [19,20]. In fact, by reversing the sign of k in the fourth-order ODE derived by Beard and Walters [11], and by augmenting the number of required boundary conditions to four by imposing the extra boundary condition at infinity, i.e., $f''(\infty) = 0$, Garg and Rajagopal [15] solved the original fourth-order ODE and observed no overshoot in the velocity profiles at any k. Interestingly, it was shown by Ariel [21] that by changing the sign of the elasticity number in the governing equation, his hybrid method enables getting results even for large values of k whereas with the same sign as used by Beard and Walters [11] his hybrid method rendered results up to $k = 0.326$ only.

From the works cited above, it can be understood that the question of the velocity in the boundary-layer overshooting its mainstream value still remains illusive. However, the above studies have demonstrated that, as far as non-Newtonian fluids are concerned, the method of solution has a profound effect on the significance of numerical results. Having said that it should be mentioned that all studies carried out thus far in relation to the stagnation point flows of viscoelastic fluids relied on the second-grade model to represent such fluids. The use of second-grade model is questionable in that this simple rheological model is good only for slow flows depicting small levels of elasticity. But there are many cases of practical interest in which the elasticity number is quite large [22]. Moreover, as mentioned above [19,20] there are some serious concerns about the sign and magnitude of model parameters appearing in a second-grade model such that the relevance of results obtained using this model is suspected even at small elasticity numbers.

Having realized the limitation of the perturbation method and also the controversies around the second-grade model (not mentioning the limitation of this rheological model to small elasticity numbers), the next step would be to rely on more realistic constitutive equations such as Maxwell, Oldroyd-B, Phan-Thien Tanner, and Giesukus [8] to simulate stagnation point flows of viscoelastic fluids. To the best of our knowledge, these rheological models have never been tried in stagnation-point flows of viscoelastic fluids. However, it should be conceded that there are works dealing with these more advanced models in other geometries. For example, Sadeghy and Sharifi [23], and Sadeghy et al. [24] have studied Blasius and Sakiadis flows of second-grade and upper-convected Maxwell models and noticed dramatic difference between their predictions regarding wall shear stress and boundary layer thickness. Similarly, Bhatnagar et al. [25] relied on Oldroyd-B model to