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A new model for the study of rain-wind-induced vibrations of a simple oscillator☆

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Abstract

In this paper, a model equation is presented for the study of rain-wind-induced vibrations of a simple oscillator. As will be shown the presence of raindrops in the wind-field may have an essential influence on the dynamic stability of the oscillator. In this model equation the influence of the variation of the mass of the oscillator due to an incoming flow of raindrops hitting the oscillator and a mass flow which is blown and shaken off is investigated. The time-varying mass is modeled by a time harmonic function whereas simultaneously also time-varying lift and drag forces are considered.

Keywords: Rain-wind-induced vibrations; Variation of the mass of the rain water; Cable stayed bridges; Bifurcation analysis

1. Introduction

Inclined stay cables of bridges are fixed on one end to a pylon and on the other end to the bridge-deck. Usually the stay cables have a polyurethane mantle and a cross-section which is nearly circular. With low structural damping of the bridge, a wind-field containing raindrops may induce vibrations of the cables.

As an example one can refer to Erasmus bridge in Rotterdam of which the stay cables vibrated heavily on November 4, 1996 less than 2 months after its opening. The problem of rain-wind-induced vibrations of stay cables has been reported and studied experimentally for the first time in [1]. Additional experimental studies can be found in [2–4]. In these papers it is remarked that regretfully calculation models are not available. A first attempt to model this problem can be found in [5] where in particular time-varying lift and drag forces are modeled. Time-varying lift and drag forces are due to the movement of the water rivulet on the cable. In

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recent years some authors [6–9] have studied models for rain-wind-induced vibrations of cables. In [7] the 2D mechanical model was introduced. On the basis of this model, the numerical investigation of the phenomenon was performed. The results obtained shows that rain-wind-induced oscillations occur on vertical cables as well. In [8], an analytical study of wind-rain-induced cable vibrations was presented by considering the influence of a moving upper rivulet on the cable. However, the varying mass of rain water on the cables has not been taken into account by these authors.

In this paper a possible additional effect is taken into account namely the variation of the quantity of rainwater located on the cable. In terms of modeling one can say that the vibrating mass of rainwater on the cable is time-dependent, and is modeled by a time harmonic function, whereas simultaneously also time-varying lift and drag forces are considered. The attention is mainly focused on the interaction of the rain with the oscillator, assuming that this interaction is an instability mechanism. Raindrops hitting the oscillator may form a rivulet or a water ridge on the oscillator. However in a stationary situation the mass flow of incoming raindrops hitting the oscillator and the mass flow of raindrops shaken off will be equal. If these mass flows are not equal then the mass of raindrops attached to the oscillator varies with time.

One may conclude that the following mechanisms may be relevant for the study of the instability of the oscillator.

- The assumption that the mass of the ridge and hence the mass of oscillator may vary in time, seems realistic
- Drag and lift forces vary usually in the dynamic situation; however, due to the fact that the position of the ridge on the oscillator is not fixed but varies with time, the aerodynamic coefficients additionally depend on time.

As the second mechanism has been studied in [5] it looks of interest to include the additional effect of time-varying mass. It should be stressed that the dynamics of the mass of the rivulets will not be modeled by a separate equation of motion in this stage: in the modeling we assume that either the position of the rivulets is fixed or varies harmonically in time in the same way as the oscillator.

2. A model equation with time-varying mass and lift and drag forces

In this section, we use the modeling principles as given in [10] or [11]. Consider a horizontal rigid cylinder with uniform circular cross section supported by springs. A rain-wind flow is directed to the axis of the cylinder. The cylinder with springs is constructed in such a way that only vertical oscillations i.e. oscillations in cross-flow are possible. The raindrops that hit the cable may stay on the surface of the cylinder for some time and may form a ridge of water of which the position varies with time. Due to the variation of the acceleration of the cable and the aerodynamic forces, part of the water will be blown and or shaken off and hence the mass of the water ridge varies in time. The system which will be studied is sketched in Fig. 1. U is the horizontal uniform velocity of the wind containing raindrops. When the cylinder moves in the positive y direction a virtual wind velocity $-\dot{y}$ is induced, i.e. a wind flow with velocity $\dot{y}$ in opposite direction. The drag force D is indicated in the direction of the resultant wind-velocity U_r, whereas the lift force L is perpendicular to D in anti clockwise direction. The water ridge on the cylinder, boldly indicated in Fig. 1, is assumed to carry out harmonic oscillations with a small amplitude on the surface of the cylinder, whereas the mass of the water ridge $m_r(t)$ is supposed to vary harmonically in time as well. The aerodynamic force F_y in vertical direction follows from Fig. 1:

$$F_y = -D \sin\phi - L \cos\phi, \tag{2.1}$$

where ϕ is the angle between U_r and U, positive in clockwise direction: $|\phi| < \pi/2$.

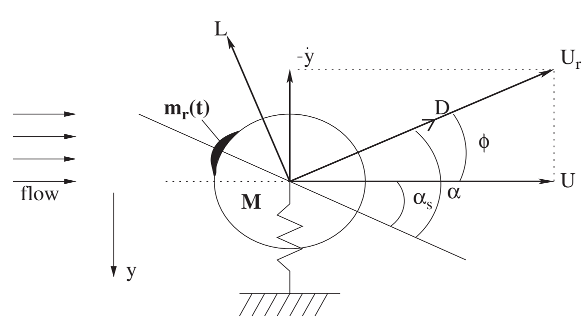

Fig. 1. Cross-section of the cylinder-spring system, fluid flow with respect to the cylinder and wind forces on the cylinder.