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Multiscale modeling of drying shrinkage and creep of concrete

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ABSTRACT

Creep and shrinkage are complicated time-dependent processes taking place in cementitious materials. In typical concretes, the most significant part of shrinkage is represented by drying shrinkage. Experimental data indicate that the ultimate value of drying shrinkage measured on concrete and mortar specimens is a nonlinear function of the ambient relative humidity. The nonlinear behavior is partially caused by microcracking, and partially by creep of C-S-H gel. On the other hand, drying shrinkage of hardened cement paste, measured on very thin specimens at a gradually decreasing relative humidity, was found to be a linear function of relative humidity. The aim of this paper is to explore whether the nonlinearities observed for macroscopic shrinkage can be explained by finite element simulations at lower levels, starting from the description of the drying cement paste by a viscoelastic model based on the Microprestress-Solidification Theory (MPS) with tensile cracking and with a constant shrinkage ratio.

1. Introduction

In civil engineering practice, the average time-dependent behavior of concrete related to the physical phenomena of creep and shrinkage is usually predicted using models recommended by design codes, e.g. *fib* Model Code 2010 [13], ACI 209 [1] and JSCE [21], or academic models, e.g. B3 [3], B4 [30], GL2000 [14] and SAK [34]. These models were developed and calibrated based on a large number of experimental results. The most recent experimental database [19], assembled at Northwestern University, contains data from 1809 shrinkage and 1403 creep experiments. All of the abovementioned models describe the drying shrinkage averaged over the section of a concrete member as

$$\varepsilon_{sh}(t) = -\varepsilon_{sh}^{\infty}\, \beta(t - t_0) \tag{1}$$

where t is the current time (measured from the first set of concrete), t_0 is the concrete age at the onset of drying, $\varepsilon_{sh}^{\infty}$ is the magnitude of ultimate shrinkage, and β is an increasing function of the duration of drying, $\hat{t} = t - t_0$, growing from 0 to 1. In all models except for JCSE and SAK, a certain parameter of function β depends on the structural (notional) size of the concrete member. The following specific formulae are used:

$$\text{B3 and B4}: \quad \beta\left(\hat{t}\right) = \tanh\sqrt{\frac{\hat{t}}{\tau_{sh}}} \tag{2}$$

$$\text{fib MC 2010 and GL2000:} \quad \beta\left(\hat{t}\right) = \sqrt{\frac{\hat{t}}{a + \hat{t}}} \tag{3}$$

$$\text{ACI:} \quad \beta\left(\hat{t}\right) = \frac{\hat{t}}{a + \hat{t}} \tag{4}$$

$$\text{JCSE and SAK:} \quad \beta\left(\hat{t}\right) = 1 - \exp -0.108\, \hat{t}^{\,0.56} \tag{5}$$

Models B3 and B4 incorporate the dependence on the member size through the shrinkage half-time τ_{sh}, and models *fib*, ACI and GL2000 use a parameter with a similar meaning, denoted here as a. Even though expressions (2)–(5) might appear to be very different, they lead to quite similar shapes of shrinkage curves; see Fig. 1a.

The magnitude of the ultimate shrinkage, $\varepsilon_{sh}^{\infty}$, is influenced by many factors; the two most important are the concrete composition and the ambient relative humidity h_{env}. The experimental data obtained from standard shrinkage experiments suggest that the resulting ultimate shrinkage strain measured on identical specimens is a strongly nonlinear function of the ambient relative humidity to which the specimens are exposed. This influence is anchored in the

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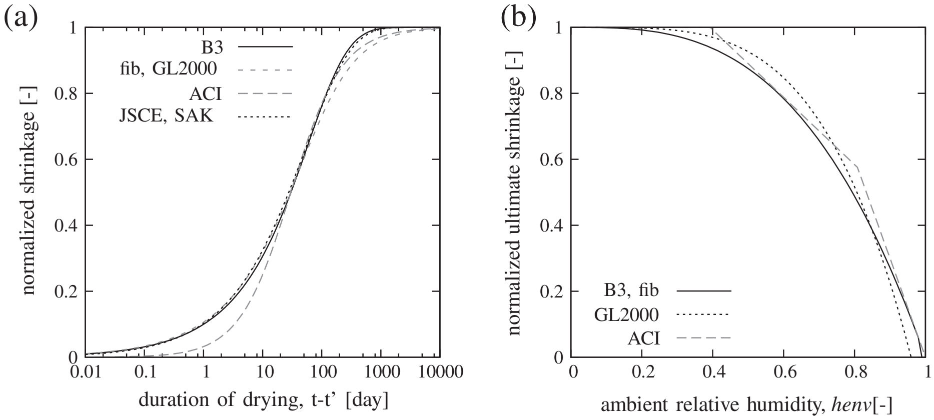

Fig. 1. Comparison of different models for concrete shrinkage: (a) function $\beta(\hat{t})$ describing evolution in time and (b) correction term k_h reflecting ambient relative humidity h_{env}.

models via a multiplicative correction factor k_h, which is expressed as follows:

$$\text{B3, B4, fib MC 2010:}\quad k_h = 1 - h_{\mathrm{env}}^3 \tag{6}$$

$$\text{GL2000:}\quad k_h = 1 - 1.18\, h_{\mathrm{env}}^4 \tag{7}$$

$$\text{ACI:}\quad k_h = \begin{cases} 1 & \ldots \text{ if } h_{\mathrm{env}} < 0.4 \\ 1.4 - 1.02\, h_{\mathrm{env}} & \ldots \text{ if } h_{\mathrm{env}} \in \langle 0.4, 0.8 \rangle \\ 3 - 3\, h_{\mathrm{env}} & \ldots \text{ if } h_{\mathrm{env}} > 0.8 \end{cases} \tag{8}$$

Since all the models have been calibrated using a similar experimental database, the basic trend should naturally be similar. As can be seen in Fig. 1b, the relation between the ambient humidity and the ultimate shrinkage is highly nonlinear, at least for relative humidities below 80%. In the Japanese models (JSCE and SAK) the correction reflecting the relative humidity of the environment is not multiplicative but additive; for this reason it is not presented here.

A closer inspection of the concrete creep and shrinkage database [19] reveals that there is only a very limited number of experiments which can directly confirm or disprove the nonlinear influence of the ambient humidity on the final shrinkage (6)–(8). The comparison of 12 selected normalized data sets with the analytical formula (6) is shown in Fig. 2a. To be included in this comparison, the shrinkage experiment had to satisfy the following requirements:

1. the specimens were subjected to at least two distinctly different levels of ambient relative humidity (lower than 98% and differing by at least 15%),
2. the tests were done using the same concrete mixture, curing procedure, age at the onset of drying, specimen size and shape,
3. the evolution of shrinkage was measured during a sufficiently long time, meaning that the measured shrinkage strain was already approaching the final horizontal asymptote in the semi-logarithmic scale.

The basic information on the selected data is summarized in Table 1. The selection should be perhaps still reduced because some of these references are outdated and the behavior of modern concrete can be different, in some cases the water-to-cement ratio was too low (which could have resulted into substantial autogenous shrinkage) or the specimens were exposed to drying too early. For the purpose of this study the experimental data from the database [19] have been adopted without further adjustments or compensations for autogenous or chemical shrinkage.

Interestingly, in a different type of experimental studies, the drying shrinkage of hardened cement paste measured on very thin specimens at a gradually decreasing relative humidity was found to be an almost linear function of the relative humidity, at least in the usual range[1] of relative humidities, as shown in Fig. 2b. This means that the origin of the nonlinear dependence of the ultimate shrinkage on humidity for concrete has to be sought in effects that arise during upscaling from the cement paste level to the concrete level.

Even though the specimens for experimental measurement of drying shrinkage are kept unloaded, internally they are not stress-free. Self-equilibrated stresses develop due to a combination of the self-restraint and the aggregate restraint. The stresses generated by the restraints are relieved by creep of the matrix and, if the tensile strength is locally exceeded, also by microcracking.

- The *self-restraint* appears as a result of differential shrinkage, whenever the cross section of the (externally unrestrained) specimen cannot shrink freely, i.e., when the distribution of relative humidity is not uniform. This effect becomes more pronounced with an increasing specimen size and with an increasing drying rate. A common consequence of this restraint is surface (micro)cracking [6].
- The *aggregate restraint* is caused by differences in shrinkage of concrete constituents, i.e., by the incompatibility between the shrinkage strains in the cement matrix and in the aggregates (the aggregates do not shrink, or shrink considerably less). The latest experimental and numerical studies [10, 15, 20] concluded that an increasing aggregate size and a decreasing volume fraction lead to a lower crack density but a larger crack opening. However, these studies were focused merely on microcracking and did not consider the influence on volume changes.

The aim of this paper is to show whether the nonlinear dependence of concrete shrinkage on ambient relative humidity can be reproduced by a meso-scale model with a linear shrinkage–humidity relationship for the hardened cement paste. The material model that

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[1] For example the *fib* Model Code 2010 [13] is applicable only for h_{env} in the range from 40% to 100%; for the evaluation of a creep coefficient it offers a simplified approach which distinguishes between dry (indoor) atmospheric conditions with $h_{\mathrm{env}} = 50\%$ and humid (outdoor) atmospheric conditions with $h_{\mathrm{env}} = 80\%$, which can be treated as representative values.