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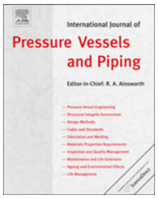

K_I–T estimations for embedded flaws in pipes – Part I: Axially oriented cracks

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ABSTRACT

This study reports a numerical investigation on the linear-elastic K_I and *T*-stress values over the front of elliptical cracks axially embedded in the wall of a pipe/cylindrical structure, under a uniform pressure applied on the inner surface of the pipe. The numerical procedure employs an interaction integral approach to compute the linear-elastic stress intensity factor (SIF) K_I and *T*-stress values from very detailed crack-front meshes. The verification study confirms the accuracy of the adopted numerical procedure in computing the K_I values based on existing results for external axial surface cracks in the wall of a cylindrical structure. The parametric investigation covers a wide range of geometric parameters including: the wall thickness to the inner radius ratio of the pipe (t/R_i), the crack depth over the wall thickness ratio (a/t), the crack aspect ratio (a/c) and the crack location measured by the ratio of the distance from the centerline of the crack to the outer surface of the pipe over the pipe wall thickness (e_M/t). Subsequent efforts develop, from a nonlinear curve-fitting procedure, a new set of equations to estimate the *T*-stress and K_I values at three critical front locations of the axial elliptical cracks: the crack-front point *O* nearest to the outer surface of the pipe, the crack-front point *I* nearest to the inner surface of the pipe and the crack-front point *M* on the centerline of the axial crack. These equations combine a second-order polynomial with a power-law expression to predict the pronounced variations in the *T*-stress and K_I values with respect to the geometric parameters. The coefficients of the new K_I and *T*-stress equations either take a constant value or incorporate the linear variation with respect to the pipe wall thickness over the inner radius ratio, t/R_i. The proposed equations demonstrate a close agreement with the finite element (FE) results, which indicate very strong dependence of the *T*-stress and K_I values at point *O* and point *I* on the corresponding ligament lengths, e_O and e_I.

1. Introduction

Cylindrical structures, *e.g.*, pipelines and pressure vessels, often embed crack-like defects during the fabrication procedure caused by lack of fusion. These defects may grow in size under internal pressures applied on the inner surface of the pipelines or pressure vessels, causing critical safety concerns to the operation of such structures. Conventional failure assessment procedures predicate predominantly on the material fracture toughness values measured from standard, through-thickness laboratory specimens (*e.g.*, compact tension specimens, C(T), or single edge-notched bend specimens, SE(B), as outlined in ASTM E-1820 [1]) with size requirements ensuring high (plasticity) constraints near the crack tip. Previous research efforts [2–5] demonstrate that the crack-front constraints, characterized by the linear-elastic *T*-stress, impose significant effects on the material fracture resistance and the crack growth process, sufficient to impact the failure assessment procedure. The *T*-stress refers to the second term in the classical Williams' solution [6] for the crack-tip stress field, and represents a uniform stress parallel to the crack plane. Wang [7] proves, from an extensive numerical investigation, that a negative (compressive) *T*-stress reduces significantly the stresses within the plastic zone ahead of the crack tip, while a positive (tensile) *T*-stress does not introduce pronounced variations in the stresses within the plastic zone. Fracture specimens with low crack-front constraints (negative *T*-stress) often yield a large plastic zone size ahead of the crack tip, while high crack-front constraints (positive or zero *T*-stress) often lead to a small-scale yielding condition. Previous researchers [8] reveal that the 2-D version of the "embedded" crack, the middle tension, M(T) specimen, experiences significantly lower crack-front constraints than does the 2-D version of the surface crack, the C(T) or SE(B) specimen. This invalidates the transferability of the K_I–T solutions obtained from a surface flaw to an embedded crack in the failure assessment procedure. Accurate evaluation of the K_I–T loading at critical front locations along the embedded flaws in the wall of a cylindrical structure becomes,

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Nomenclature

C_i	Coefficients to be determined from the nonlinear curve-fitting procedure
D_i	Coefficients to be determined from the nonlinear curve-fitting procedure
E	Elastic modulus
I	Interaction integral
J	Energy release rate
K_I	Mode I stress intensity factor
L	Half-length of the pipe
N	Number of elements along the half-elliptical crack front
N'	Average number of elements per crack-front length a
P	Perimeter of an ellipse
P_{ij}	First Piola–Kirchhoff stress tensor
R_i	Inner radius of the pipe
R_m	Radius of the pipe measured at the mid-thickness of the wall
R_o	Outer radius of the pipe
T	*T*-stress
V_0	Volume of the domain surrounding the crack tip
W	Strain energy density
Y_i	*i*th data point
a	Crack depth
c	Half-crack length
e_I	Remaining ligament length at crack-front point I
e_M	Distance from the crack-front point M to the outer surface of the pipe
e_O	Remaining ligament length at crack-front point O
f	Line load applied at the apex of a wedge
f_i	Linear or quadratic polynomials in terms of a/t, a/c and e_i/t ($i = O$, I or M)
g_i	Linear or quadratic polynomials in terms of a/t, a/c and e_i/t ($i = O$, I or M)
h	Expected parametric function
m	Number of equations
p_i	Pressure applied on the inner surface of the pipe
q	Weighting function
r	Radius
s	Position along the crack front
t	Wall thickness of the pipe
u_i	Displacement vector
(X_1, X_2, X_3)	Cartesian coordinate system with the origin at the crack tip
(x, y, z)	Cartesian coordinate system with the origin at the center of the cylinder
ε_{ij}	Strain tensor
ϕ	Crack-front angle
χ^2	The chi-square term
$\overline{\eta}$	A vector of parameters
θ	Angle
σ_{ij}	Cauchy stress tensor
σ_i	Square root of the variance
σ_∞	Remote stress
υ	Poisson's ratio

therefore, essential to ensure reliable integrity assessment of such structures.

Recent investigations prove that the sign of *T*-stresses affects both the growth of a stationary crack under static loads and the propagation of a fatigue crack under cyclic loads. The works by Tvergaard [2] and Tvergaard and Hutchinson [3] show that a negative *T*-stress increases substantially the material fracture resistance, while a positive *T*-stress introduces a very slight variation in the fracture resistance. In contrast, Fett and Munz [9] confirm the observation by Cotterell and Rice [10] that a positive *T*-stress causes the crack path to deviate from the original crack plane while a negative *T*-stress shows a negligible effect on the crack path. Jayadevan et al. [11] conclude, from an extensive numerical investigation, that both the sign and magnitude of *T*-stresses impose significant effects on the crack growth under dynamic loading conditions. Roychowdhury and Dodds [12] demonstrate that the *T*-stress varies the plasticity-induced crack closure for a small-scale yielding regime under cyclic loading. Yin et al. [13] address the applicability of the two-parameter K_I–T approach in assessing the cleavage fracture failure for reactor pressure vessels under thermal–hydraulic loading conditions.

The last two decades observe substantial developments in numerical methods to compute the linear-elastic *T*-stress values for cracks in engineering structures. Nakamura and Parks [14] introduce the interaction integral approach in computing the elastic *T*-stress along a 3-D crack front. This approach becomes a widely adopted method in computing mixed-mode stress intensity factors (SIF) and *T*-stresses in many engineering problems [15,16]. Chen and Lin [17] and Chen et al. [18,19] utilize the eigenfunction expansion variational method to compute the elastic *T*-stress in cracked plate-type structures. Chen et al. [20] adopted a *p*-version finite element method to improve the convergence and accuracy of the numerical *T*-stress solutions. Some researchers [21,22] coupled the boundary element method and the finite element method to compute *T*-stresses under both static and dynamic loading conditions. Yang and Ravi-Chandar [23] proposed a new approach to compute the *T*-stress by the difference between two stress components σ_{11} and σ_{22}. Jayadevan et al. [24] incorporated the line-spring elements in their computation of *T*-stress values for surface cracks in pipelines.

Recent research efforts focused on the development of *T*-stress solutions for cracks in different fracture specimens and structural configurations under various loading conditions, through analytical and numerical approaches. Chen [25] derives the closed form *T*-stress solutions for a few plane elasticity crack problems using the complex potential theory. Kirilyuk and Levchuk [26] presented the analytical *T*-stress solution for an embedded elliptical crack in an infinite elastic body under remote tension and bending, and for the same crack under linearly varying pressures applied on the crack surfaces [27]. Molla-Abbasi and Schutte [28] derived the analytical *T*-stress results for internal elliptical cracks in an infinite medium under mixed-mode loading conditions through a superposition method. The German research group [29,30] describes their numerical implementation of the analytical *T*-stress solutions for an edge crack in a semi-infinite space and for a crack loaded by near-tip stresses. A few researchers [31–39] have addressed the *T*-stress problem for different 3-D crack configurations in finite thickness plates and standard fracture specimens. For cylindrical structures, *T*-stress solutions are available for a few limited crack configuration and loading conditions including: through-wall cracks under general loading condition [40], multiple edge cracks in thick-walled cylinders [41], circumferential cracks in cylinders [42], longitudinal surface cracks in thick-walled cylinders [43] and edge cracks in thick-walled cylinders [44].

The current study aims to provide simple estimates on the K_I and *T*-stresses at critical locations of the axially embedded crack