International Journal of Pressure Vessels and Piping 86 (2009) 280–289

Contents lists available at ScienceDirect

International Journal of Pressure Vessels and Piping

journal homepage: www.elsevier.com/locate/ijpvp

A nonlinear Hermitian transfinite element method for transient behavior analysis of hollow functionally graded cylinders with temperature-dependent materials under thermo-mechanical loads

M. Shariyat*

Faculty of Mechanical Engineering, K.N. Toosi University of Technology, Tehran, Iran

A R T I C L E I N F O

Article history:
Received 24 April 2008
Received in revised form 18 August 2008
Accepted 4 September 2008

Keywords:
Nonlinear transfinite element
Thermoelastic stresses
Wave propagation
Hermitian elements
FGM
Temperature-dependency
Thick-walled cylinder
Numerical Laplace inversion

A B S T R A C T

In the present paper, an algorithm for nonlinear transient behavior analysis of thick functionally graded cylindrical vessels or pipes with temperature-dependent material properties under thermo-mechanical loads is presented. In contrast to researches presented so far, a Hermitian transfinite element method is proposed to improve the accuracy and to prevent artificial interference or cohesion formation at the mutual boundaries of the elements. Time variations of the temperatures, displacements, and stresses are obtained through a numerical Laplace inversion. Another novelty of the present research is using the transfinite element method to solve nonlinear problems. A sensitivity analysis includes investigating effects of the volume fraction index, dimensions, and temperature-dependency of the material properties is performed. Results confirm the efficiency of the present algorithm and reveal the significant effects of the temperature-dependency of the material properties and the elastic wave reflections and interferences on the responses. In comparison to other techniques, the present technique may be used to obtain relatively accurate and stable results in a less computational time.

1. Introduction

Components made of functionally graded materials (FGMs) are generally constructed to sustain elevated temperatures and severe temperature gradients. Low thermal conductivity, low coefficients of thermal expansion and core ductility have enabled the functionally graded materials to withstand higher thermal and mechanical shocks. Continuously varying the volume fraction of the mixture in the FGMs eliminates the interface problems, mitigates thermal stress concentrations, and causes a more smooth stress distribution [1,2].

Among various FGM structures, cylindrical components have been of special interest. Transient heat transfer analysis is a vital stage in the development of strength investigations such as dynamic thermal buckling, fatigue life assessment under cyclic thermal loads, dynamic crack propagation, etc. Majority of the well-known heat transfer analyses performed so far for thick FGM cylinders are generally restricted to uniform heating [3] or steady-state heat transfer analyses [4–9]. Some of these researches have been accomplished based on the multi-layer discretization approximation [7].

Some authors have investigated the transient heat transfer in isotropic cylinders. Using a finite Hankel transform, Wang [10] studied the transient thermoelastic behavior of a hollow cylinder subjected to a rapid arbitrary heating. Kandil et al. [11] presented a numerical method for thermal stress analysis of a thick-walled cylinder under dynamic internal temperature rise. The transient response of a thick-walled pipe subjected to a general internal temperature excitation was studied by Segall [12,13]. Lee [14] presented a thermoelastic analysis for multi-layered cylinders under periodic loading conditions. Shahani and Nabavi [15] investigated thermoelastic behavior of a thick-walled cylinder analytically using the finite Hankel transform. Ramadan [16] proposed a semi-analytical solution procedure for transient heat transfer analysis in multi-layered media.

Limited researches have been presented for transient heat transfer analysis of the FGM cylinders. Furthermore, almost all of the presented researches have ignored the effect of the temperature-dependency of the material properties. Reddy and Chin [17] and Praveen et al. [18] have developed finite element formulations to analyze the pseudo-dynamic thermoelastic responses of functionally graded cylinders subjected to abrupt thermal loadings. Obata et al. [19] analyzed the two-dimensional unsteady

* Tel.: +98 9122727199; fax: +98 21 88674748.
E-mail addresses: m_shariyat@yahoo.com, shariyat@kntu.ac.ir

0308-0161/$ – see front matter © 2008 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ijpvp.2008.09.004

thermal stresses in an FGM hollow circular cylinder by the Laplace transformation method. Using the finite-difference method, Awaji and Sivakumar [20] analyzed the steady-state and transient temperature distributions in an FGM cylinder. Based on the multi-layered cylinder approximation, Kim and Noda [21] studied axisymmetric two-dimensional transient thermoelasticity of an infinite hollow FGM circular cylinder using Green's function approach. Based on the local boundary integral equations with moving least square approximation of the temperature and the heat flux, and using the numerical Laplace inversion method, Sladek et al. [22] studied the transient heat conduction in FGM cylinders. Wang et al. [23,24] used the first-order finite element method in conjunction with the finite-difference method to study the one-dimensional transient heat conduction. Hosseini et al. [25] and Shao et al. [26,27] employed analytical methods to study the transient conduction heat transfer in FGM cylinders with material properties that follow an exponential law. Recently, thermo-mechanical analysis of an FGM hollow circular cylinder subjected to a linearly increasing boundary temperature is developed by Shao and Ma [28]. Thermo-mechanical properties of the functionally graded materials were assumed to be temperature independent. A Laplace transform technique and a series method were employed to solve ordinary differential equations of FGM cylinders with material properties that follow an exponential constitutive law.

Almost in all of the above mentioned researches, the temperature-dependency of the material properties was neglected. Recently a transient thermal analysis taking the temperature-dependency of the material properties into account is introduced by the author [29]. Time integration and updating methods were used.

Heyliger and Jilania [30] adopted a variational method to study frequency response of inhomogeneous cylinders and spheres. Steinberg [31] formulated the inverse spectral problem to determine properties of a cylinder with inhomogeneous materials. Han et al. [32] presented a hybrid numerical method for analysis of transient waves in an FGM cylinder. The displacement responses were determined by employing the Fourier transformations. El-Raheb [33] studied effects of the circumferential and the radial inhomogeneities on the transient waves of a hollow cylinder. The cylinder was divided into isotropic sub-cylinders. The static–dynamic superposition method is employed to determine the transient response. Shakeri et al. [34] studied vibration and transient behaviors of FGM thick hollow cylinders subjected to axisymmetric dynamic loads using a first-order Galerkin finite element and a time integration method. The functionally graded cylinder was divided into isotropic sub-cylinders. In each interface between two layers, stress and displacement continuity conditions were satisfied through modifying the generalized Hooke's law. Ponnusamy [35] discussed the wave propagation in a generalized thermoelastic isotropic solid cylinder with arbitrary cross-section. The asymmetric transient response of a hollow cylinder containing a compressible fluid was analyzed by El-Raheb [36].

Majority of the finite element formulations presented in the heat transfer, thermoelastic wave propagation or elastic wave propagation fields to date are based on the Lagrangian shape functions [17,18,23,24,34,37–39]. Since only C^0 continuity is guaranteed by Lagrangian elements, stress components experience jumps at the mutual boundaries of the elements [40–42], especially when linear elements are used. Therefore, artificial sources of concentrated tractions or wave sources will form at the mutual boundaries of the elements. Generally, in such cases, simultaneous incorporation of the displacement and the stress continuity conditions may lead to some contradictions, such as altering the generalized Hooke's law at the mutual boundaries of the elements [40,41]. Although the idea of dividing the FGM cylinder into some isotropic sub-cylinders is commonly used, it may induce successive wave reflections and may affect the results. Bruck [43] and Samadhiya et al. [44] have proved that the elastic wave propagation in discretely layered FGMs is somewhat different. The transmitted wave and the reflected waves from each sharp interface between the discrete layers may affect the stress distribution.

In the present paper, a nonlinear transient response analysis of FGM thick cylinders subjected to thermo-mechanical loading conditions is presented taking the temperature-dependency of the material properties into consideration. In the present paper, in contrast to works presented so far, the transfinite element method is employed to solve a nonlinear problem. It is known that Laplace and Fourier integral transforms are applicable to linear systems, only. Therefore, the three main novelties of the present research are development of a procedure for solving nonlinear transient problems by the numerical transfinite element method, incorporating the temperature-dependency of the material properties in a thermoelastic analysis, and using Hermitian elements to improve the accuracy of the results and to ensure that both displacement and stress components are continuous at the mutual boundaries of the elements. The resulting highly nonlinear governing equations are solved using a numerical scheme that is based on simultaneous numerical Laplace inversion, successive approximations, and updating. Finally, results based on the assumptions of temperature-dependency (TD) and temperature-independency (TID) of the material properties are compared with each other and effects of various parameters on the transient responses are studied.

2. The governing equations

Geometric parameters of the thick-walled FGM cylinder are shown in Fig. 1. Variation of the material properties in terms of the temperature may be expressed as follows [45]:

$$P = P_0\left(1 + P_1T + P_2T^2 + P_3T^3\right) \tag{1}$$

where P_0, P_1, P_2 and P_3 are some material constants. The FGM cylinder is assumed to be made of a mixture of two constituent materials. If the inner layer ($r = r_i$) of the cylinder is ceramic-rich, whereas the external layer ($r = r_o$) is metal-rich, the material properties of the FGM cylinder at any arbitrary point through the thickness may be expressed as

$$P = P_m + (P_c - P_m)\left(\frac{r_o - r}{r_o - r_i}\right)^n \tag{2}$$

c and m subscripts denote the ceramic and the metal properties, respectively. n is the so-called volume fraction index. Therefore, from Eqs. (1) and (2) one may conclude that

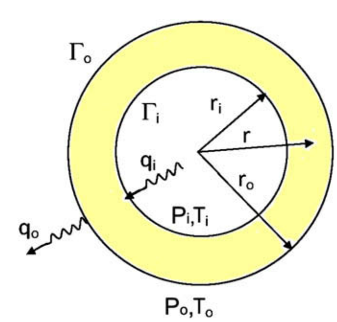

Fig. 1. Geometric parameters of the considered thick-walled FGM cylinder.