available at www.sciencedirect.com

ScienceDirect

journal homepage: www.elsevier.com/locate/ijrefrig

Lévy statistics and beta model: A new solution of "wall" turbulence with a critical phenomenon

Peter W. Egolf[*,1]

University of Applied Sciences of Western Switzerland, Hesso, Institute of Thermal Science and Engineering, IGT CH 1401 Yverdon-les-Bains, Switzerland

ARTICLE INFO

Article history:
Received 14 January 2009
Received in revised form
13 April 2009
Accepted 2 May 2009
Published online 14 May 2009

Keywords:
Turbulent flow
Fluid
Boundary layer
Wall
Research
Modelling

ABSTRACT

By applying statistical mechanics Brown's motion leads to the macroscopic diffusion equation, and similar Lévy flights and the β model are the specific flux origin to derive the Difference-Quotient Turbulence Model (DQTM). It is applied to a turbulent flow in the overlap region close to a wall and leads to a mean velocity deficit power law, which may serve as a substitute for the "logarithmic law" with the von Karman constant. Its exponent is slightly dependent on the stress parameter (e.g. the Reynolds number) and is confirmed by the Princeton super pipe data. The solution of this fluid dynamic standard example reveals a physical order concept, just as known in the theory of critical phenomena.

Statistiques de Lévy et modèle béta : nouvelle solution pour la turbulence proche de la paroi avec un phénomène critique

Mots clés : Écoulement turbulent ; Fluide ; Couche limite ; Paroi ; Recherche ; Modélisation

1. Introduction

Turbulent flows occur in nature and engineering wherever one looks. The earth's atmosphere including the jet stream, tornados and winds, the oceans with the Gulf Stream, tsunamis, lakes and rivers, they all show turbulent phenomena. In engineering the flows over the wing of an aircraft, over cars, ships, etc. are usually turbulent. In refrigeration systems turbulence occurs in gas and liquid flows in pipes, after expansion valves, in heat exchangers, mixing

* Tel.: +41 24 557 61 57; fax: +41 24 557 75 79.
E-mail address: peter.egolf@heig-vd.ch
[1] Member of IIR commission B2.

0140-7007/$ – see front matter © 2009 Elsevier Ltd and IIR. All rights reserved.
doi:10.1016/j.ijrefrig.2009.05.001

Nomenclature

Symbols

a	basic value for probability
a	distance (m)
a	constant
A	constant
b	constant
b	step size of a Lévy walk (m)
b	width (m)
B	constant
c	constant
C	constant
d	space dimension
D	fractal dimension
e	spectral energy density ($J\,m^{-2}$)
E	specific turbulent kinetic energy ($J\,m^{-3}$)
f	general function
h	scaling exponent (Hurst exponent)
j	specific flux ($m\,s^{-1}$)
k	wave number (m^{-1})
k	exponent
l	eddy diameter (m)
m	number of created eddies
m	intermediate eddy generation number
m	exponent
M	total number of eddies
M	total specific momentum ($kg\,m^{-2}\,s^{-1}$)
M	dimensionless magnetization
n	number of generation
n	integer number
N	number related to largest Lévy step
o	occupation number
O	occupation number of all eddies
O	order parameter
p	probability distribution (–)
p	integer number
p	exponent
p	specific eddy momentum ($kg\,m^{-2}\,s^{-1}$)
p	production of turbulent kinetic energy ($J\,m^{-3}$)
P	pressure ($Pa\,m^{-2}$)
q	integer number
q	occupation probability
Re	Reynolds number
s	general step size (m)
s	exponent
s	spin value
s	standard deviation (m)
S	stress parameter
t	time (s)
t	eddy turnover time (s)
T	eddy presence time (s)
T	transition region
u	eddy turning velocity ($m\,s^{-1}$)
U	maximum velocity in a cross section ($m\,s^{-1}$)
u	velocity ($m\,s^{-1}$)
v	velocity ($m\,s^{-1}$)
x	space coordinate (m)
y	space coordinate (m)

Greek symbols

α	constant
α	stress parameter
β	constant
β	space filling factor
β	order parameter
χ	distance (m)
δ	Kronecker symbol
δ	width of boundary layer (m)
ϵ	energy transfer rate ($W\,m^{-3}$)
Δ	difference
Δt	eddy generation time (s)
η	dimensionless coordinate
κ	auxiliary variable
λ	general constant
λ	structure function
ν	viscosity ($m^2\,s^{-1}$)
ρ	density ($kg\,m^{-3}$)
σ	constant
τ	eddy creation time interval (s)
τ	shear stress ($N\,m^{-2}$)
ω	constant
Ω	auxiliary variable (dep., e.g. $m^{-2/3}$)
ξ	auxiliary variable

Subscripts

active	active region of eddies in space
channel	value for a channel
crit	critical value
eddy	idealized fluid dynamic object
liquid	liquid phase
max	maximal value
min	minimal value
pole	pole value
solid	solid phase
wall	value at a wall
0	largest eddy
0	maximal (at leading edge)
1	downstream
11, 22	normal
2	perpendicular to main flow direction
21	shear

Superscripts

*	general variable
*	friction variable
′	fluctuation quantity
+	dimensionless (inner variable)
$\overline{\cdots}$	mean value (time averaged)
↑	spin up
↓	spin down