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Transversely isotropic hyperelastic-viscoplastic model for glassy polymers with application to additive manufactured photopolymers

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ABSTRACT

A typical feature of additive manufactured photopolymers is that their mechanical behavior depends on the printing direction. Hence, this work aims at developing a material model to predict the inelastic deformation and failure of glassy polymers with such an effect. To achieve this goal, a transversely isotropic hyperelastic-viscoplastic model is proposed, which considers the effects of strain rate, pressure, temperature, and printing direction. In addition, a modified Tsai-Wu failure criterion is proposed to predict the macroscopic failure of glassy polymers with strain softening. The proposed model is applied to simulate the deformation and failure of additive manufactured lattice structures. Experimental and simulation results indicate that the mechanical behavior of additive manufactured lattice structures depends on not only the lattice orientation but also the printing direction.

1. Introduction

Photopolymerization is one of the major technologies used for the additive manufacturing (AM) of functional polymer components (de Obaldia et al., 2015; Dimas et al., 2013; Kang et al., 2014; Zhang et al., 2015a). The representative techniques include stereolithography (SLA), PolyJet, and multiphoton lithography, which fuse the monomers and oligomers together by using UV light or laser layer by layer. These techniques usually produce parts with high resolution but low distortion. For example, the SLA and PolyJet techniques can readily achieve resolution on the order of tens of microns. In addition, the state-of-the-art SLA and multiphoton lithography techniques can even manufacture structures in micro- and nano-scales (Meza et al., 2014; Zheng et al., 2014). However, a critical and common issue of these techniques is that the photopolymer component exhibits strong printing direction effect inherited from the layer-wise processing feature. In addition, mechanical testing results show that the printing direction will not only affect the deformation but also the strength of the photopolymers (Blanco et al., 2014; Cazón et al., 2014). Therefore, there is a strong demand to develop advanced material models to characterize the printing direction effect and predict the mechanical response and failure of these AM polymeric structures.

The inelastic deformation of glassy polymers usually undergoes initial yielding, strain softening, and subsequent hardening (Boyce et al., 1988; Sudarkodi and Basu, 2014). In early years, a seminal model of glassy polymers was developed by Parks et al. (1984) to describe these features and was later on generalized by Boyce et al. (1988) to include the effects of strain

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rate, pressure, and temperature. After that, this framework was further developed by a variety of researchers. For example, a notable contribution was the adoption of the eight-chain model (Arruda and Boyce, 1993a, b) to characterize the backstress evolution, which has a relatively simple form but good accuracy compared to experimental results. In addition, a different 1D rheological model was introduced by Bergström and Boyce (1998), which is also quite popular nowadays to model the rate dependent inelastic behavior of elastomers. Most of these inelastic polymer models adopt the eight-chain model (Arruda and Boyce, 1993a, b) to characterize the hyperelastic behavior of the elongated polymer chains, while some researchers also tried other polymer chain network models to evaluate the backstress (Miehe et al., 2009; Tomita and Tanaka, 1995; Wu and Van der Giessen, 1993). Another important contribution to the modeling of glassy polymers was due to Anand and Gurtin (2003), who established a thermodynamic framework and introduced an internal state variable to capture the nonlinear stress-strain behavior. Thereafter, extensive research has been conducted on the thermo-mechanical coupling models for glassy polymers to consider temperature induced material softening and glass transition (Anand et al., 2009; Arruda et al., 1995; Belbachir et al., 2010; Dupaix and Boyce, 2007; Mulliken and Boyce, 2006; Richeton et al., 2007; Srivastava et al., 2010; Varghese and Batra, 2009). In addition, some researchers also applied or modified these models for special polymers such as shape memory polymers (Qi et al., 2008), photodegradable polymers (Belbachir et al., 2010), photopolymers (Wang et al., 2011) among others. Besides the modeling of glassy polymers, research effort has also been devoted to semicrystalline polymers. For example, some physics-based models for semicrystalline polymers were proposed by Boyce et al. (2000) and Ahzi et al. (2003) and further developed in a number of works (Ayoub et al., 2011, 2010; Makradi et al., 2005; Popa et al., 2014; Uchida and Tada, 2013). Meanwhile, some researchers also proposed phenomenalogical models for semicrystalline polymers (Colak, 2005; Drozdov, 2008; Drozdov and Gupta, 2003; Dusunceli and Colak, 2008; Regrain et al., 2009). These models for semicrystalline polymers will not be introduced in details since the focus of this work is on glassy polymers. Although numerous models have been proposed for glassy polymers, these models still suffer from some limitations when applied to AM photopolymers. On the one hand, these models are usually devised for isotropic glassy polymers, e.g. isotropic elastic tensor, von Mises stress, and isotropic hyperelastic model are used, which do not consider the material anisotropy induced by the printing direction effect. On the other hand, these models usually adopt an associated flow rule, which leads to unphysical volume dilatation when the material is pressure sensitive (Nemat-Nasser, 2004). Therefore, one aim of the present work is to develop a transversely isotropic inelastic model for photopolymers to tackle these two critical problems, which has improved accuracy compared with the isotropic model used for photopolymers (Wang et al., 2011).

The AM photopolymers usually show orientation-dependent failure behavior, that is, the interface between two printing layers is usually weaker than the intra-layer strength under tensile loading. A macroscopic failure criterion is useful for engineering analysis to estimate the material and structure failure (Christensen, 2013). Some representative stress-based failure criteria are, for instance, the Tsai-Wu criterion (Tsai and Wu, 1971) and Hashin criterion (Hashin, 1980), which were originally developed for fiber composites and have been widely used. However, these stress-based failure criteria are not applicable when the material exhibits strain softening, in which case one stress value may correspond to several strain values. In contrast, the failure criteria (Feng, 1991; Volokh, 2013) formulated in the strain space can overcome this issue, but they are difficult to extend to problems involving inelastic deformation since the strain is decomposed into elastic and inelastic parts. Therefore, the stress-based formulation is adopted in this work by modifying the Tsai-Wu criterion to handle the failure problems with strain softening. Note that a well-developed macroscopic failure criterion is quite useful for engineering failure analysis, and indeed there is still a lack of such model for photopolymers.

The goal of this work is to model the orientation-dependent inelastic deformation and failure of AM photopolymers informed by the experimental data. In order to achieve this goal, a transversely isotropic hyperelastic-viscoplastic model is established by considering the effects of material anisotropy, pressure sensitivity, and strain rate, and a failure criterion is proposed by modifying the Tsai-Wu model so it is applicable to the strain softening cases. Finally, the developed material model and failure criterion are implemented into the user subroutine (VUMAT) of the finite element software package ABAQUS to simulate the structural response and failure of lattice structures. The proposed model can also be applied to analyze the deformation and failure of other transversely isotropic glassy polymers and polymer composites.

2. Kinematics of finite deformation

The inelastic deformation of the glassy polymers is studied in the finite deformation scenario. The deformation of a continuum body is illustrated in Fig. 1. The finite strain deformation of this deformed body is described by the deformation gradient $\mathbf{F}$, which maps a material point $\mathbf{X}$ of the reference configuration $\mathscr{B}_0 \subset \mathbb{R}^3$ in the 3D Euclidean space $\mathbb{R}^3$ to a spatial point $\mathbf{x}(\mathbf{X})$ in the current configuration $\mathscr{B} \subset \mathbb{R}^3$, as

$$\mathbf{F} = \frac{\partial \mathbf{x}}{\partial \mathbf{X}} \tag{1}$$

The corresponding velocity gradient, $\mathbf{L}$, is given by

$$\mathbf{L} = \frac{\partial \dot{\mathbf{x}}}{\partial \mathbf{x}} = \dot{\mathbf{F}}\mathbf{F}^{-1} = \mathbf{D} + \mathbf{W} \tag{2}$$