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# On fracture in finite strain gradient plasticity

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ABSTRACT

In this work a general framework for damage and fracture assessment including the effect of strain gradients is provided. Both mechanism-based and phenomenological strain gradient plasticity (SGP) theories are implemented numerically using finite deformation theory and crack tip fields are investigated. Differences and similarities between the two approaches within continuum SGP modeling are highlighted and discussed. Local strain hardening promoted by geometrically necessary dislocations (GNDs) in the vicinity of the crack leads to much higher stresses, relative to classical plasticity predictions. These differences increase significantly when large strains are taken into account, as a consequence of the contribution of strain gradients to the work hardening of the material. The magnitude of stress elevation at the crack tip and the distance ahead of the crack where GNDs significantly alter the stress distributions are quantified. The SGP dominated zone extends over meaningful physical lengths that could embrace the critical distance of several damage mechanisms, being particularly relevant for hydrogen assisted cracking models. A major role of a certain length parameter is observed in the multiple parameter version of the phenomenological SGP theory. Since this also dominates the mechanics of indentation testing, results suggest that length parameters characteristic of mode I fracture should be inferred from nanoindentation.



## 1. Introduction

Experiments and direct dislocation simulations have shown that metallic materials display strong size effects at the micron and sub-micron scales. Attributed to geometrically necessary dislocations (GNDs) associated with non-uniform plastic deformation, this size effect is especially significant in fracture problems as the plastic zone adjacent to the crack tip may be physically small and contains large spatial gradients of deformation.

Much research has been devoted to modeling experimentally observed size effects (e.g., Fleck and Hutchinson, 1993; Niordson and Hutchinson, 2003a; Bardella, 2010; Klusemann et al., 2013) and several continuum strain gradient plasticity (SGP) theories have been proposed through the years in order to incorporate length scale parameters in the constitutive equations. Of particular interest from the crack tip characterization perspective is the development of formulations within the finite deformation framework (e.g., Gurtin and Anand, 2005; Gurtin, 2008; Polizzotto, 2009). In spite of the numerical complexities associated, various studies of size effects under large strains have been conducted using both crystal (Kuroda and Tvergaard, 2008; Bargmann et al., 2014) and isotropic (Niordson and Redanz, 2004; Legarth, 2007; McBride and Reddy, 2009;

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Anand et al., 2012) gradient-enhanced plasticity theories. Isotropic SGP formulations can be classified according to different criteria, one distinguishing between phenomenological theories (Fleck and Hutchinson, 1997, 2001) and microstructurally/mechanism-based ones (Gao et al., 1999; Qiu et al., 2003).

The experimental observation of cleavage fracture in the presence of significant plastic flow (Elssner et al., 1994; Korn et al., 2002) has encouraged significant interest in the role of the plastic strain gradient in fracture and damage assessment. Studies conducted in the framework of phenomenological (Wei and Hutchinson, 1997; Komaragiri et al., 2008; Nielsen et al., 2012) and mechanism-based theories (Wei and Xu, 2005; Siddiq et al., 2007) have shown that GNDs near the crack tip promote local strain hardening and lead to a much higher stress level as compared with classical plasticity predictions. However, although large deformations take place in the vicinity of the crack, the aforementioned studies were conducted within the infinitesimal deformation theory and little work has been done to investigate crack tip fields modeled by SGP accounting for finite strains. Hwang et al. (2003) developed a finite deformation framework for the mechanism-based strain gradient (MSG) plasticity theory but were unable to reach strain levels higher than 10% near the crack tip due to convergence problems. Pan and Yuan (2011) used the element-free Galerkin method to characterize crack tip fields through a lower order gradient plasticity (LGP) model (Yuan and Chen, 2000). From a phenomenological perspective, Tvergaard and Niordson (2008) analyzed the influence of the strain gradient at a crack tip interacting with a number of voids while Mikkelsen and Goutianos (2009) determined the range of material length scales where a full strain gradient dependent plasticity simulation is necessary.

Very recently, Martínez-Pañeda and Betegón (2015) identified and quantified the relation between material parameters and the physical length over which gradient effects prominently enhance crack tip stresses from a mechanism-based approach. The numerical results obtained in Martínez-Pañeda and Betegón (2015) show a significant increase in the differences between the stress fields of MSG and conventional plasticity when finite strains are taken into account. This is due to the strain gradient contribution to the work hardening of the material, which lowers crack tip blunting and thereby suppresses the local stress triaxiality reduction characteristic of conventional plasticity predictions (McMeeking, 1977). These results revealed the important influence of strain gradients on a wide range of fracture problems, being particularly relevant in hydrogen assisted cracking modeling due to the central role that the stress field close to the crack tip plays on both hydrogen diffusion and interface decohesion. Moreover, Gangloff and his co-workers have shown that accurate correlations with experimental measurements can be achieved by adopting high levels of hydrostatic stress from dislocation-based micromechanical modeling of hydrogen embrittlement (Thomas et al., 2003; Lee and Gangloff, 2007; Gangloff et al., 2014).

In this paper crack tip fields are evaluated thoroughly with both phenomenological and mechanism-based strain gradient plasticity theories with the aim of gaining insight into the role of the increased dislocation density associated with large gradients in plastic strain near the crack. Differences between the two main classes of SGP theories are examined and their physical implications discussed. In both approaches the numerical scheme is developed to allow for large strains and rotations providing an appropriate framework for damage and fracture assessment within SGP theories.

## 2. Material models

The key elements of the two SGP theories considered in this work are summarized in this section, with particular emphasis on the constitutive equations and other aspects of interest from the fracture mechanics perspective. Comprehensive details, including the variational formulation and the corresponding differential equations, can be found in Fleck and Hutchinson (2001), Niordson and Hutchinson (2003a), Gao et al. (1999), Qiu et al. (2003) for the phenomenological and mechanism-based cases, respectively.

### *2.1. Fleck and Hutchinson's gradient theory*

The strain gradient generalization of $J_2$ flow theory proposed by Fleck and Hutchinson (2001) is considered to model size effects in metal plasticity from a phenomenological perspective. In this theory hardening effects due to plastic strain gradients are included through the gradient of the plastic strain rate $\dot{\varepsilon}^p_{ij,k} = (m_{ij}\dot{\varepsilon}^p)_{,k}$. Where $\dot{\varepsilon}^p = \sqrt{\frac{2}{3}\dot{\varepsilon}^p_{ij}\dot{\varepsilon}^p_{ij}}$ is the increment in the conventional measure of the effective plastic strain and $m_{ij} = 3/2s_{ij}/\sigma_e$ is the direction of the plastic strain increment, with $s_{ij}$ denoting the stress deviator, and $\sigma_e$ the von Mises effective stress. The gradient enhanced effective plastic strain rate, $\dot{E}^p$ can be defined in terms of three unique, non-negative invariants of $\dot{\varepsilon}^p_{ij,k}$, which are homogeneous of degree two:

$$\dot{E}_p = \sqrt{\dot{\varepsilon}^{p^2} + l_1^2 I_1 + l_2^2 I_2 + l_3^2 I_3} \tag{1}$$

where, $l_1$, $l_2$ and $l_3$ are material length parameters. The effective plastic strain rate can be expressed explicitly in terms of $\dot{\varepsilon}^p$ and $\dot{\varepsilon}^p_{,i}$:

$$\dot{E}_p = \sqrt{\dot{\varepsilon}^{p^2} + A_{ij}\dot{\varepsilon}^p_{,i}\dot{\varepsilon}^p_{,j} + B_i\dot{\varepsilon}^p_{,i}\dot{\varepsilon}^p + C\dot{\varepsilon}^{p^2}} \tag{2}$$

where the coefficients $A_{ij}$, $B_i$ and $C$ depend on the three material length parameters as well as on the spatial gradients of the plastic strain increment direction (for details see Fleck and Hutchinson, 2001).