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A general anisotropic yield criterion for pressure-dependent materials

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ABSTRACT

Many engineering materials exhibit strong anisotropy along with a pressure dependence on the plastic yield behavior. The ability to predict the effects of pressure dependence and plastic anisotropy of these materials on their respective yield surfaces is important for accurately analyzing their behavior during deformation. The current work develops a general anisotropic yield criterion that includes the capability of modeling pressure-dependent effects either additively or multiplicatively in the yield criterion at the onset and during the evolution of plastic deformation. The developed yield criterion can be used to represent both quadratic and non-quadratic yield surfaces and has no restrictions on the symmetricity of the plastic behavior, allowing the entire spectrum from isotropic to fully anisotropic. The pressure dependence of the yield criterion and the corresponding plastic rate of deformation equation are expressed as general functional forms to allow development and incorporation of new pressure dependence functions or to accept existing pressure dependence functions directly. Therefore, the developed yield criterion can be used as a framework for accurately modeling the plastic behavior of various material systems, largely reducing the complexity in yield surface description for different classes of materials and allowing a systematic approach for implementation in numerical analysis procedures such as the finite element method. Two existing pressure-dependent models are reformulated to show the applicability of the general yield criterion framework presented herein to developing new non-quadratic anisotropic models with a dependence on pressure. While the general yield criterion is developed as an extension of a specific non-quadratic anisotropic yield criterion, the same model development methodology can be applied to other novel or existing yield criteria that are defined in the deviatoric stress space.

1. Introduction

Development of a yield criterion for plastically deformable materials in finite element analysis is of ample importance for accurately depicting the evolution of plastic state variables and the overall material behavior. The mathematical concept of the yield criterion greatly simplifies the computational aspects of analysis for crystalline materials, which would otherwise likely use the computationally demanding crystal plasticity modeling approach. However, there is certainly no restriction for applying the yield criterion concept only to crystalline materials as it has been used for describing composite (Camanho et al.,

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2013; Vogler et al., 2013), cementitious (Yu et al., 2010), geomechanical (Chakraborty et al., 2013; Lai et al., 2009; Yang et al., 2010; Yuanming et al., 2010), and polymeric (Caddell et al., 1973; Ghorbel, 2008) material classes as well. The evolution of plastic state variables for a given material class can be quite unique to its class, thus, deciding either which yield criterion to use or how to develop a new yield criterion may not be obvious. The present work is directed at establishing a general yield criterion that can be easily amended to include material sensitivity to multiple states of stress, e.g., deviatoric, hydrostatic, and triaxial, and that can be utilized for arbitrary symmetricity of the plastic behavior.

Two of the most widely used yield criteria developed for crystalline materials are those of von Mises (1913) and Tresca (1864). In the von Mises yield criterion, it is assumed that the material will evolve to a new state of plasticity when the second invariant (J_2) of the Cauchy stress deviator reaches a critical value. The Tresca yield criterion simply compares the maximum state of shear stress in the material with a critical value to determine whether plastic yield will occur. These two models are often taken as the foundation of models extended to include other mechanics such as a pressure or stress triaxiality dependence. For example, the Drucker–Prager yield criterion (Drucker and Prager, 1952) is an extension of the von Mises yield criterion to include pressure dependence via the first invariant of the Cauchy stress (I_1) in an additive fashion. Materials that are inherently pressure-dependent include those previously mentioned and also some that are perhaps less obvious, e.g., pharmaceutical materials (Han et al., 2008), showing the impact of these models on the general science community.

Although the von Mises and Tresca yield criteria can be used effectively for some materials, it has been observed (Stout et al., 1983) that there are limitations to these models because the yield surface for many materials should be represented with a higher degree of nonlinearity than quadratic, as proposed by von Mises, but not so far as to reach that proposed by Tresca, which is nonlinear to the highest degree. An attempt to model materials such as these was given initially by both Hershey (1954) and Hosford (1972). The Hershey/Hosford model is an extension of the von Mises yield criterion for arbitrary nonlinearity. This model added some generality to the yield criterion; however, the resulting yield criterion is pressure-independent and thus is limited to materials that are assumed to be plastically insensitive to this particular stress state.

Recently, rigorous upscaling methodologies have been used by Cazacu et al. (2014) and Revil-Baudard and Cazacu (2014) in order to describe yielding of porous solids where the matrix obeys the Tresca criterion. These works are developed using a strain-rate potential basis for evolution of the state of plasticity in the material. In contrast to existing heuristic yield criteria, the new porous Tresca criterion involves coupling between shear and mean stress effects, the yield locus being centro-symmetric. The 3D extension of the porous Tresca model indicated that the shape of the yield locus in the octahedral plane depends strongly on the mean stress, evolving from a hexagon with rounded corners for low levels of mean stress to a triangle with rounded corners for higher levels of mean stress. These works also provide a very thorough study of the sensitivity of porous solids to the stress invariants for general multiaxial load states.

It has also been observed that most materials exhibit, and in some cases are designed to exhibit (Blanc et al., 2006), anisotropic plastic behavior. Hill (1948), Hosford (1972), and Barlat et al. (1991) developed some of the earlier models specifically for analyzing this behavior. However, Hill 1948 criterion was developed with a limitation in that the yield surface was again quadratic in nature. This lack of generality was circumvented by both Barlat and Hosford, allowing the yield surface to be of arbitrary nonlinearity and anisotropic. These models were again sensitive only to the state of shear in the material. The Barlat et al. (1991) model was an extension of Hosford's model where the yield function was evaluated using a linear transformation of the Cauchy stress tensor such that a resultant yield surface could be evaluated and the stresses could be updated during plastic deformation using the transformed stress and then simply converted back to the actual Cauchy stress. The linear transformation of the Cauchy stress tensor has since been employed to incorporate anisotropic behavior in many yield criteria (Barlat et al., 1997, 2003, 2005, 2007; Bron and Besson, 2004; Dunand et al., 2012; Yoon et al., 2014; Yoshida et al., 2013).

An elegant approach was presented by Karafillis and Boyce (1993) that utilized the benefits of the aforementioned anisotropic models, e.g., the linear stress transformation and the arbitrary nonlinearity, while attempting to remove the shortcomings, i.e., the lack of both general dimensional stress states and general anisotropic behavior. The Karafillis–Boyce model (KB93) incorporates a linear stress transformation tensor of the Cauchy stress into an Isotropic Plasticity Equivalent (IPE) deviatoric stress, later incorporated in other anisotropic model developments (e.g., Barlat et al. (1997), Maniatty et al. (1999)), that is used to evaluate a yield criterion that is a weighted mixture of two yield surfaces. The two yield surfaces used for the resultant surface are such that the lower bound converges to the Tresca yield criterion and the upper bound converges to that described by Hershey and Hosford. The resultant surface can also be set to the von Mises yield criterion when the nonlinearity is quadratic in nature. The yield surface mixing combined with the linear stress transformation leads to an extremely general modeling approach that can be tailored for a vast number of materials and material classes so long as they are characterized by shear-dominated plastic behavior. Successful utilization of this model, specifically for metal forming applications, can be found in works by Cao et al. (2000), Yao and Cao (2002), and Korkolis and Kyriakides (2008). However, this model lacks the ability to describe the pressure dependence of materials in that the IPE stress is a deviatoric tensor.

Cazacu and Barlat (2001) developed a generalization of Drucker's yield criterion (Drucker, 1949) to orthotropy by first creating a method for extension of any isotropic yield function, which is expressed in terms of the second and third invariants of the Cauchy stress deviator (J_2 and J_3, respectively) for orthotropic plasticity. While in the present work J_3 will not be formally considered, the generalization used in the work by Cazacu and Barlat is an excellent example of an attempt to create a procedure for extension of material models to include orthotropic behavior based on the linear transformation tensor presented by Barlat et al. (1991) and Karafillis and Boyce (1993). Likewise, the present work will utilize the linear