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Strain rate tensors and constitutive equations of inelastic micropolar materials

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ABSTRACT

Nonlinear micropolar continuum model allows to describe complex micro-structured media, for example, polycrystals, foams, cellular solids, lattices, masonries, particle assemblies, magnetic rheological fluids, liquid crystals, etc., for which the rotational degrees of freedom of material particles are important. The constitutive equations of the hyperelastic nonlinear micropolar continuum can be expressed using the strain energy density depending on two strain measures. In the case of inelastic behavior the constitutive equations of the micropolar continuum have more complicated structure, the stress and couple stress tensors as well as other quantities depend on the history of strain measures. In what follows we discuss the constitutive equations of the nonlinear micropolar continuum using strain rates.

1. Introduction

Recently the interest to generalized models of continuum mechanics is growing with respect to the necessity to describe of media with microstructure and more complex behavior of materials. Among the generalized media there are strain-gradient, micropolar (Cosserat) and micromorphic models of continua. In particular, the model of micropolar or Cosserat continuum is used for the description of such micro-structured media as, for example, polycrystals, foams, cellular solids, lattices, masonries, particle assemblies, soils, magnetic rheological fluids, liquid crystals, etc., for which the rotational degrees of freedom of material particles and the couple stresses are important.

Specific but very perspective for engineering are so-called nanocrystalline materials investigated over the past couple of decades, see Gleiter (2000) and Meyers et al. (2006). The latter are ultrafine-grained materials with a grain size under 100 nm with special properties. So these materials contain an extremely large fraction of grain boundaries with properties different from the ones of bulk materials. In other words, such microstructure of nanostructured materials is determined by characteristic length scale of order of few or tenth nanometers. Among various methods of manufacturing of nanostructured materials are electrodeposition, crystallization, and after severe plastic deformations, see Valiev et al. (2000) and Valiev (2004). Another example of nanostructured materials are metal nanolaminates, see Mara et al. (2004, 2008), Misra and Gibala (2000) and Misra et al. (2004). There are thousands of papers that appeared on this topic since few last decades. One of the peculiarities of nanostructured materials is the size-effect that is dependence of material properties on the

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microstructure characteristic length. Modeling of the elasto-plastic behavior of nanostructured materials is based on various approaches, see review by Gates et al., 2005 on multiscale simulations. The influence of non-homogeneities including the grain-size effect on the material properties discussed by Valiev et al. (2000), Valiev (2004), Meyers et al. (2006), Berbenni et al. (2007), McDowell (2008, 2010) and Misra and Singh (2014).

McDowell (2008, 2010) and Luscher et al. (2010) highlighted that for description of such non-classical materials some extended theories of plasticity can be applied that is the micropolar, micromorphic, strain gradient plasticity theories. The micropolar theory of plasticity begin since the works by Sawczuk (1967) and Lippmann (1969). Lippmann (1969) has published paper entitled "A Cosserat Theory of Plastic Flow" where he generalized the flow rule taking into account the couple stresses, see also further discussion on possible applications by Becker and Lippmann (1977), Bogdanova-Bontcheva and Lippmann (1975) and Diepolder et al. (2001) and the review Lippmann (1995). The idea of using micropolar kinematics for plastic flow is based, in particular, on the possibility to take into account independent grain or other subdomains rotations. As in classical theories of plasticity, the micropolar plasticity under small and finite deformations can be developed with the use of multiplicative or/and additive decompositions, see Vardoulakis (1989), de Borst (1993), Steinmann (1994a,b), Forest et al. (2000), Forest and Sievert (2006), Grammenoudis and Tsakmakis (2001, 2005a,b, 2009), Neff (2006) and Salehi and Salehi (2014). Development of the micropolar plasticity was highly motivated by proper description of strain localization observed during plastic deformations, see original paper by de Borst (1991), Dietsche et al. (1993), Perić et al. (1994), Ehlers and Volk (1997), Dietsche and Willam (1997), Forest (1998), Forest et al. (2001) and Bauer et al. (2012). Keller and Trusov (2002) suggested incremental model with independent spin for description of plastic deformations. Mathematical study of problems of the micropolar elastoplasticity are performed by Neff and Chełmiński (2005, 2007), Chełmiński and Neff (2008) and Neff et al. (2007, 2009).

Description of plastic deformations of nanostructured materials within the framework of inelastic Cosserat continuum is similar to modeling of behavior of granular and porous media given by Bogdanova-Bontcheva and Lippmann (1975), Becker and Lippmann (1977), Ehlers and Volk (1997), Mori et al. (1998) and Matsushima et al. (2003), of discrete structures such as masonries, see Trovalusci and Masiani (1997), Ehlers et al. (2003), Masiani and Trovalusci (1996), Besdo (2010) and Trovalusci and Pau (2013). The Cosserat plasticity is strongly related with strain gradient plasticity and theory of dislocations, see Sievert et al. (1998), Gurtin (2002), Forest and Sedláček (2003), Forest and Sievert (2003), Forest (2008), Clayton et al. (2006), Besson et al. (2010), Clayton (2011) and Polizzotto (2014). In particular, Mayeur and McDowell (2014) recently presented comparison between gradient plasticity of Gurtin-type and micropolar plasticity under small deformations. For strain gradient plasticity we refer also to the landscape works by Mühlhaus and Aifantis (1991), Fleck and Hutchinson (1997), Gao et al. (1999), Huang et al. (2000), Gurtin (2002) and Bertram and Forest (2013). The strain gradient theories find widely applications in other fields of mechanics, in elasticity (Aifantis, 2003; dell'Isola et al., 2009b), poromechanics (dell'Isola et al., 2000; Sciarra et al., 2007, 2008), for description capillarity-related phenomena (dell'Isola and Seppecher, 1997, 1995; dell'Isola et al., 2009a, 2012), and even for bone remodeling (Madeo et al., 2012). The extended models of continua such as the second-gradient models require extended boundary conditions considered for example by dell'Isola and Seppecher (1995), dell'Isola et al. (2012), Srinivasa and Reddy (2013) and Baek and Srinivasa (2003).

The micropolar plasticity can be considered as a part of more wide micromorphic theory of plasticity for which microstrains are related not only with microrotations but with complete microdeformations, see Eringen (1999) and Forest (2013). The micromorphic plasticity is presented by Forest and Sievert (2006), Forest (2009), Grammenoudis et al. (2009), Grammenoudis and Tsakmakis (2010), Regueiro (2010), Forest et al. (2014). Since the micropolar and micromorphic theories can be considered as theories with internal degrees of freedom it is worth to mention the works on the inelasticity theories based on internal variables by Rice (1971), Maugin and Muschik (1994), Hirschberger and Steinmann (2009) and Horstemeyer and Bammann (2010). Indeed, the microrotation tensor used in the micropolar continuum can be interpreted as a tensor-valued internal variable describing the rotational degrees of freedom of nano-sized grains, particles in suspensions, etc., while the balance of angular momentum as a corresponding balance equation for such internal variable, see Eringen (1999) and Capriz (1989).

For the proper formulation of the constitutive equations one has to formulate strain measures and corresponding strain rates. Within the framework of classical Cauchy continuum the discussion of strain measures and strain rates is presented in thousands of papers, see for example Xiao et al. (1997b), Bruhns et al. (1999), Xiao et al. (1997a, 1998b,a, 2000) and Surana et al. (2013) and references therein. The comparison of stress-rate-type of constitutive equations is given by Szabó and Balla (1989). The specific logarithmic strain rate and the corresponding constitutive equations were introduced by Xiao et al. (1997b,a). A review on the state of the art is given in the recent paper by Bruhns (2014).

The discussion of strain measures for the polar elastic materials is presented by Pietraszkiewicz and Eremeyev (2009a,b), where the natural Lagrangian and Eulerian strain measures are introduced as a result of three possible ways, that from pure geometrical considerations, from the requirement of invariance of the strain energy density under superposed rigid body motions and as measures conjugate to corresponding stress tensors. Ramezani and Naghdabadi (2007) discussed strain–stress pairs within the framework of micropolar mechanics. Using strain rate tensors Ramezani and Naghdabadi (2010) and Ramezani et al. (2008) consider incremental equations of the micropolar hypo-elasticity. Trovalusci and Masiani (1997) established the interrelations between strain rates in discrete and continual models.

The aim of the paper is to present the foundations of a constitutive theory for nonlinear micropolar inelastic media. For that purpose we extend in a systematic way the material modeling approach settled by Noll and Trusdell to Cosserat