International Journal of Plasticity 63 (2014) 124–137

Contents lists available at ScienceDirect

International Journal of Plasticity

journal homepage: www.elsevier.com/locate/ijplas

Limit state of structures made of heterogeneous materials

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ARTICLE INFO

Article history:
Received 26 February 2014
Available online 14 May 2014

Keywords:
Limit state
Plasticity
Heterogeneous materials
Finite elements method
Optimization

ABSTRACT

A numerical approach is presented to determine the load bearing capacity of structural elements made of heterogeneous materials subjected to variable loads. Melan's lower-bound shakedown theorem is applied to representative volume elements. Combined with the homogenization technique, the material effective properties are determined through transformation from the mesoscopic to macroscopic admissible loading domains. For the numerical applications, finite element method and large-scale nonlinear optimization, based on an interior-point-algorithm, are used. The methodology is illustrated by the application to regular and random heterogeneous materials. This way, the proposed method provides a direct numerical approach to evaluate the macroscopic strength of heterogeneous structures as a useful tool for the design of structures.

1. Introduction

Structural elements made of heterogeneous materials are widely used in civil and mechanical engineering with increasing demands on security and sustainability and to guaranty their integrity as a necessary condition for safe functioning is most important. The characterization of the overall material properties of composites has therefore attracted much attention over the last decades and a large body of research has been performed to obtain the effective material properties of heterogeneous media through various multi-scale modeling techniques (see e.g. Nemat-Nasser and Hori, 1993; Ponte-Castañeda and Suquet, 1998; Michel et al., 2001; Pierard et al., 2007; McDowell, 2010; Dunant et al., 2013).

If the material is loaded beyond the elastic limit with non-deterministic loading history, standard simulation methods fail or can be highly inefficient. To solve this problem, so-called Direct Methods (DM), subsuming shakedown and limit analysis, are proposed in this paper. In particular we discuss how to determine the global effective material properties, taking into account the characteristics of the material microstructures, including size, morphology, strength and distribution of heterogeneities.

This methodological choice is justified by the fact that DM does not need any information about the loading path (König, 1987). For this reason, the application of DM to heterogeneous materials has found great interest over the last decade. Lower-bound methods (e.g. Weichert et al., 1999; Magoariec et al., 2004; Zhang et al., 2004; Makrodimopoulos and Martin, 2006; Spagnoli et al., 2014), upper-bound methods (e.g. Ponter and Leckie, 1998; Carvelli et al., 1999; Capsoni et al., 2001; Li and Yu, 2006; Makrodimopoulos and Martin, 2007; Li, 2011) and dual methods (Tirosh, 1998; Pisano et al., 2014) have been applied to study mostly periodic heterogeneous materials, with regular geometrical arrangement of constituents.

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http://dx.doi.org/10.1016/j.ijplas.2014.03.019
0749-6419/

The aim of the present paper is to extend the authors' former approach (Chen et al., 2013b) to heterogeneous materials with random microstructure and to propose a new methodology for optimal design of structures made of heterogeneous materials. We investigate how to determine the elastic and plastic homogenized parameters of heterogeneous materials in order to provide data for the design of structures: elastic material parameters are obtained basing on the homogenization theory and the constitutive laws, whereas the plastic material parameters are predicted by using yield surface fitting on obtained admissible macroscopic homogenized stress domains.

For materials with random microstructure, it is unlike to construct a single RVE to study the global material behavior: due to the difference of morphology among samples, despite having identical size and constituents, the scatter of their associated mechanical behavior is still remarkable. To overcome this difficulty, an algorithm is developed which automatically converts scanning electron microscopy (SEM) images into RVEs. Then, the lower-bound approach of DM is performed on a group of RVEs built from the real microstructure and their results are interpreted statistically.

The lower-bound method leads to a nonlinear optimization problem, and thus the good choice of an efficient optimization algorithm is important. Therefore a special software package has been especially developed by the authors (Akoa et al., 2007; Simon and Weichert, 2011) based on the interior point method. Through the analysis of local stress and strain, the influence of each phase can be evaluated and using homogenization theory, the global response of heterogeneous material is obtained.

The proposed method is illustrated by new results using three-dimensional finite elements taking into account multi-dimensional loading domains. In order to validate the proposed procedure for random microstructures, conventional incremental analyses are performed on selected samples for comparison.

2. Shakedown analyses of heterogeneous materials

2.1. Lower-bound analysis

The lower-bound approach used in this paper is based on Melan's shakedown theorem (Melan, 1938); limit analysis is treated as particular case of shakedown analysis. Melan's theorem can be formulated for elastic-perfectly plastic material as follows: If there exist a safety factor $\alpha > 1$ and a time-independent residual stresses field $\bar{\boldsymbol{\rho}}(\boldsymbol{x})$, which, superposed with the purely elastic stresses $\boldsymbol{\sigma}^E$ does not exceed the yield condition F at any time $t > 0$,

$$F(\alpha\boldsymbol{\sigma}^E + \bar{\boldsymbol{\rho}}, \sigma_Y) \leqslant 0 \tag{1}$$

with

$$\text{Div } \bar{\boldsymbol{\rho}} = 0 \text{ in } V \tag{2}$$

$$\mathbf{n} \cdot \bar{\boldsymbol{\rho}} = 0 \text{ on } \partial V \tag{3}$$

then shakedown takes place under arbitrary load paths contained within a given load domain $\boldsymbol{\mathcal{L}}$. Here, $\boldsymbol{\sigma}^E$ is the field of the purely elastic stresses satisfying the equilibrium and boundary conditions for given external loading, σ_Y is the yield stress, $\mathbf{n}$ is the outer normal on the boundary ∂V. The lower bound approach described above suggests that the problem of shakedown analysis may be implemented in the following two steps: construction of the field of purely elastic stresses $\boldsymbol{\sigma}^E$ for all possible independent loads and solution of the constrained optimization problem to determine the loading factor α and the field of the residual stresses $\bar{\boldsymbol{\rho}}$.

2.2. Homogenization method

The discussion of the effective properties of heterogeneous material with periodic microstructures is similar to the more general discussion of the effective properties of random media, except that the information on the microstructure is complete in the periodic case, whereas it is only partial in the random case. The question of the effective properties of heterogeneous material implicitly assumes that the problem contains two scales which can be separated. The microscopic scale (local scale) is small enough for the heterogeneities to be separately identified. The macroscopic scale (overall scale) is large enough, compared to the microscopic scale, for the heterogeneities to be smeared-out. The homogenization method describes the relation between these two scales mainly by two stages: localization and globalization. Any macroscopic point in a heterogeneous structure is investigated in a RVE. This process is termed localization. The inverse procedure, by which mesoscopic properties originating in the unit cell are idealized at the macroscopic level, is called globalization.

With $\boldsymbol{x}$ and ξ as the global and local coordinates, respectively, the following relationship holds:

$$\xi = \frac{\boldsymbol{x}}{\epsilon} \tag{4}$$

ϵ is a small scale parameter, which determines the size of the RVE. It plays an important role in studying the heterogeneous material, especially for non-uniform structures. There has been much research about how to determine a representative volume element. The various classes and definitions of RVE and the main practical approaches can be referred to in the review of (Pelissou et al., 2009). Although here only periodic heterogeneous materials are investigated, also a RVE with random spatial distribution can be considered through the "Window" technique (Shan and Gokhale, 2002). For a heterogeneous material