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On the thermodynamically consistent modeling of distortional hardening: A novel generalized framework

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ABSTRACT

Many important physical effects of materials undergoing plasticity at the macroscale cannot be captured realistically by isotropic and kinematic hardening only. For instance, the evolution of the texture in polycrystals results macroscopically in a distorted yield surface. This paper deals with adequate hardening models for such a distortion. To be more precise, a novel general frame for finite strain plasticity models is elaborated. To the best knowledge of the authors, it is the first one combining the following features: (1) proof of thermodynamical consistency; (2) decomposition of distortional hardening into dynamic hardening (due to currently active dislocations) and latent hardening (due to currently inactive dislocations); (3) difference of the yield surface's curvature in loading direction and in the opposite direction. The cornerstone of this model is a new plastic potential for the evolution equations governing distortional hardening. Although this type of hardening is characterized through a fourth-order tensor as internal variable, the structure of the aforementioned potential is surprisingly simple. Even though the final model is rather complex, it requires only few model parameters. For these parameters, in turn, physically sound bounds based on the convexity condition of the yield surface can be derived. Three different examples demonstrate the predictive capabilities of the novel framework.

1. Introduction

Although a certain trend towards a micromechanical description of polycrystals can indeed be observed in the literature, cf. e.g. Homayonifar and Mosler, 2012; Miehe et al., 2002, macroscopic models are still most frequently applied, if problems at the technologically most relevant scale are to be analyzed, (see Nebebe et al., 2009; Mekonen et al., 2012) and references cited therein. The reason for this is their numerical efficiency. However, this positive feature is accompanied with a high complexity of the underlying constitutive model. To be more precise and focusing on polycrystals, the macroscopic model has to capture effects (in the sense of homogenization theory) due to the rotation of the atomic lattice within the individual grains as well as the distortion (e.g., elongation) of the grains. In many cases that cannot be realized by isotropic and kinematic hardening models only, since the aforementioned effects often lead to a distortion of the macroscopic yield surface. Models accounting for this transformation are referred to as *distortional hardening models*. A broad variety of these approaches can be found, e.g., in Baltov and Sawczuk (1965), Ortiz and Popov (1983), Haddadi et al. (2006), Feigenbaum

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and Dafalias (2007), Feigenbaum and Dafalias (2008), Barthel et al. (2008), Noman et al. (2010), Shi and Mosler (2013). Most of these approaches are based on evolution equations for a fourth-order tensor describing the plastic material symmetry of the material, cf. Baltov and Sawczuk, 1965; Dafalias, 1979; Ortiz and Popov, 1983; Feigenbaum and Dafalias, 2007; Barthel et al., 2008; Noman et al., 2010.

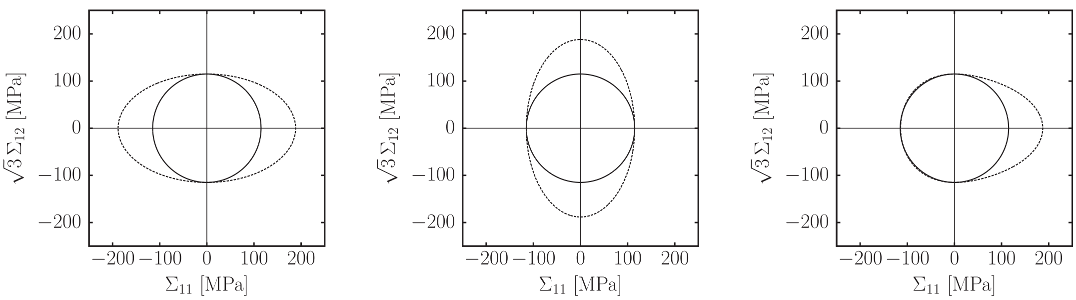
A comprehensive overview and comparison of some distortional hardening models are presented in Shi and Mosler (2013). According to this paper, most models share one similarity: distortional hardening is governed by an Armstrong–Frederick-type evolution equation. Despite this fundamental similarity, the models are significantly different, and therefore their range of applications also differs. In order to explain these differences, the most important physical effects and their influence on the macroscopic yield surface are summarized in Fig. 1. In all figures, a uniaxial tension test is considered. Fig. 1a shows the typical dynamic hardening effect. At the microscale, currently active dislocations lead to this phenomenon at the macroscale. Depending on the material, the opposite effect, i.e., dynamic softening, can also arise. In addition to an elongation/shrinkage of the yield surface in loading direction, the orthogonal direction can also undergo (cross) hardening/softening. This is depicted in Fig. 1b. Particularly for materials showing a pronounced variation of the Lankford coefficient (r-value), an uncoupling of dynamic and latent hardening is important. Physically speaking, latent hardening is due to currently inactive dislocations which have to be crossed by the active ones and thereby yield additional hardening. Finally, for some materials such as high-strength aluminum alloys, a higher curvature of the yield function is observed in loading direction in contrast to the reverse direction. This special type of dynamic hardening is outlined in Fig. 1c. To the best knowledge of the authors, only one of the existing constitutive models in the literature covers all of the aforementioned phenomena, cf. Pietryga et al., 2012. However, thermodynamical consistency of this models is not proven.

Within the models advocated in Baltov and Sawczuk (1965), Dafalias (1979), Ortiz and Popov (1983), Feigenbaum and Dafalias (2007), Feigenbaum and Dafalias (2008), Shi and Mosler (2013), dynamic and latent hardening are captured by means of only one evolution equation. Although this simplicity is indeed appealing, such an approach is usually too restrictive for real materials. To be more precise, the implied coupling between latent and dynamic hardening does not agree with the results observed in many experiments.

Models accounting for a different response of dynamic and latent hardening are discussed in Peeters et al. (2002), Haddadi et al. (2006), Barthel et al. (2008), Noman et al. (2010). Conceptually, two independent evolution equations governing dynamic and latent hardening are introduced for that purpose. By doing so, the so-called *cross hardening effect* can be captured more realistically. Although the cited models do improve the predictive capabilities of the cross hardening effect, thermodynamically consistency has not been proven yet. Furthermore, the stress space implied by the evolution equations in Barthel et al. (2008) and Noman et al. (2010) can be unbounded, resulting in stresses of infinite magnitude. These effects will be analyzed in detail in the present paper.

Within the third class of distortional hardening models, a higher curvature of the yield function in loading direction compared to the reverse direction is incorporated, cf. Ortiz and Popov, 1983; Feigenbaum and Dafalias, 2007; Feigenbaum and Dafalias, 2008. This is realized by coupling distortional and kinematic hardening. Such a coupling is important for the modeling of high-strength aluminum alloys for instance.

The concise review of existing distortional hardening models shows that a model capturing the distortion of the yield surface due to latent and dynamic hardening for which thermodynamical consistency is explicitly shown, is still missing. Such a missing unified framework is elaborated in the present paper. This framework falls into the range of so-called *generalized standard materials* in the sense of Mandel (1971) and Lemaitre (1985) and thus, the second law of thermodynamics is automatically fulfilled. Furthermore, the effect of a higher curvature of the yield surface in loading direction can be incorporated directly into this framework. In this respect, the novel model unifies the approach proposed in Feigenbaum and Dafalias (2007) and Feigenbaum and Dafalias (2008) and that suggested in Barthel et al. (2008) and Noman et al. (2010). Physical bounds of the model parameters related to distortional hardening are derived by analyzing convexity of the saturated yield surface.



(a) dynamic hardening

(b) latent hardening

(c) dynamic hardening with high curvature along loading direction

Fig. 1. Sketch of yield surface evolution with distortional hardening.