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Thermo-coupled elastoplasticity models with asymptotic loss of the material strength

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ABSTRACT

Rate-independent finite elastoplastic equations with thermo-coupled effects are proposed to bypass the yield condition and loading–unloading conditions. These new equations are shown to be more realistic and of much simpler structure than classical equations. In a sense of thermodynamic consistency the strength property of elastoplastic solids is then studied from the standpoint of stress-bearing capacity. It is shown that asymptotic loss of the strength may be derived directly from elastoplastic equations, thus leading to the finding of a noticeable phenomenon, namely, asymptotic vanishing of the stress concurrent with developing elastoplastic flow. It is indicated that this finding may suggest a natural constitutive characterization of fatigue, fracture and failure as certain limiting cases of elastoplastic behavior. Simple models with asymptotic loss of the strength are constructed as examples of potential practical applications

1. Motivation and introduction

The load-bearing capacity of an engineering structure rests on the thermomechanical behavior of the applied materials in response to various loading conditions. Adequate and practical representations of thermomechanical behaviors of materials under various loading conditions are accordingly of central, substantial importance. In fact, it is at the heart of modern continuum mechanics and theory of materials as well as related engineering fields. In the framework of continuum mechanics, fundamental quantities representing thermomechanical responses of a deformable body include the deformation, the stress, the temperature and the heat flux, etc. Of them, the deformation quantity represents continuing shape changes of the material body from a kinematic standpoint, while the stress is the macroscopic characterization of the internal resistant reactions of the material body to experienced deformations due to the internal interactions characteristic of this material. Moreover, the temperature and the heat flux characterize thermal effects with dissipation in courses of motion and deformation. Then, the thermomechanical behavior of a material body is modeled by certain constitutive relations prescribing how the stress and the heat flux are related to deformation and temperature as well as their histories. Alongside certain universal physical laws common to all kinds of materials, constitutive relations of material behavior, also known as constitutive models of materials, play a central role in analyzing and assessing significant mechanical problems of materials and structures subjected to typical loadings and actions.

Of particular interest is the assessment of reliability and safety problems associated with fatigue, fracture and failure, etc. In recent years, numerous studies have been made from various standpoints. Here, only certain representatives of most recent results are mentioned below. Stoughton (2000) presented a general forming limit criterion for sheet metals with

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Nomenclature

$\boldsymbol{F}$	the deformation gradient
J	the volumetric ratio; $J = \det \boldsymbol{F}$
$\boldsymbol{L}$	the velocity gradient; $\boldsymbol{L} = \dot{\boldsymbol{F}}\boldsymbol{F}^{-1}$
$\boldsymbol{D}$	the stretching; $\boldsymbol{D} = \frac{1}{2}(\boldsymbol{L} + \boldsymbol{L}^T)$
$\boldsymbol{\sigma}$	the Cauchy stress
$\boldsymbol{\tau}$	the Kirchhoff stress; $\boldsymbol{\tau} = J\boldsymbol{\sigma}$
$\tilde{\boldsymbol{\tau}}$	the deviatoric part of $\boldsymbol{\tau}$
$\boldsymbol{T}$	the absolute temperature
$\boldsymbol{q}$	the heat flux vector
ζ	the heat supply
ψ	the free energy per unit reference volume
η	the entropy per unit reference volume
$\mathcal{D}$	the intrinsic dissipation (see (3))
$\boldsymbol{H}$	the heat conductivity tensor
$\bar{W}$	complementary thermo-elastic potential (see Eq. (8))
$\boldsymbol{D}^e$	the elastic stretching
$\boldsymbol{D}^p$	the plastic stretching
θ	the plastic work (see (23))
ϑ	the effective plastic work (see Eq. (14))
$\boldsymbol{\alpha}$	the back stress
$\overset{\circ}{\boldsymbol{\tau}}{}^{\log}$	the logarithmic rate of $\boldsymbol{\tau}$
$\overset{\circ}{\boldsymbol{\alpha}}{}^{\log}$	the logarithmic rate of $\boldsymbol{\alpha}$
$\mathbb{S}$	the elastic stiffness tensor (see Eq. (13))
$\mathbb{H}$	hardening tensor (see Eqs. (15) and (17))
ρ	the plastic indicator (see Eq. (41))
β	the plastic index (see Eqs. (41) and (42))
f	the plastic characteristic function (see Eq. (28))
r	the stress limit (see Eqs. (28) and (29))
g	Effective stress norm (see Eqs. (28)–(30))
c	Prager's modulus (see Eqs. (18) and (19))
ω	the hysteresis modulus (see Eqs. (18) and (19))
$\hat{h}$	the plastic modulus (see Eq. (39))
$\check{h}$	the normalized plastic modulus (see Eq. (40))
$\hat{f}$	the stress-rate loading function (see Eqs. (12) and (37))
$\check{f}$	the strain-rate loading function (see Eqs. (12) and (38))
Υ	the loading factor (see Eq. (35) or Eq. (36))
$\psi_0(T)$	the specific heat (see Eq. (21))
$\varphi(\vartheta, T)$	increasing function characterizing free energy (see Eq. (24))
r_0	the initial stress limit (see Eq. (51))
$s,\ a$	positive dimensionless constants (see Eq. (51))
ϑ_m	the extremum point of the stress limit (see Eq. (51))
$\lambda,\ c_0,\ \omega_0$	hardening parameters (see Eq. (65) and (66))
$E,\ \nu$	Young's modulus, Poisson ratio
$\frac{\partial f}{\partial \mathbf{A}}$	2nd-order gradient tensor with Cartesian components $\frac{\partial f}{\partial A_{ij}}$
$\frac{\partial^2 \bar{W}}{\partial \boldsymbol{\tau}^2}$	4th-order gradient tensor with Cartesian components $\frac{\partial^2 \bar{W}}{\partial \tau_{ij} \partial \tau_{kl}}$
$\mathbb{S}^{ep}$	the elastoplastic rigidity tensor (see Eqs. (48) and (49)

an ultimate split in the sheet and Huang et al. (2000) proposed a criterion for failure prediction in anisotropic sheet metals. A review in this respect is given by Stoughton and Zhu (2004). Subsequent analyses and developments may be found, e.g., in Wu et al. (2005) for forming limit analysis, Bronkhorst et al. (2006) for a study of the localization behavior of tantalum and stainless steel, Shyam et al. (2007) for a model of small fatigue crack growth in metallic materials, Yoshida et al. (2007) and Benzerga et al. (2012) for the path-dependence property of the forming limit and the fracture locus, Stoughton and Yoon (2005, 2011, 2012) for anisotropic materials under non-proportional loading and for path-independent forming limits etc., Kim et al. (2011) for a study of the shear fracture, Jansen et al. (2013) for an anisotropic stress-based criterion to predict the fracture mechanism etc., and, in particular, Khan and Liu (2012a,b) for latest advances concerning strain rate and temperature effects. On the other hand, studies have been made based on continuum damage mechanics. Recent results in this