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Necking limit of substrate-supported metal layers under biaxial in-plane loading

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A R T I C L E I N F O

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Necking instability often indicates the onset of ductile failure. It has been shown that the necking instability in a substrate-supported metal layer can be retarded to a higher strain than that in a single freestanding metal layer. Most existing theoretical studies of the necking limit of substrate-supported metal layers assume plane strain condition. However, most commonly conducted experiments of such metal/substrate bilayers are uniaxial tensile tests. So far, the necking instability of substrate-supported metal layers under arbitrary combinations of biaxial in-plane loading conditions remains poorly understood. This paper presents a comprehensive study of the necking limit of a metal/substrate bilayer over the full range of biaxial loading ratio, from 1 for equibiaxial loading, to 0 for plane strain loading, and to $-1/2$ for uniaxial loading. Two representative material combinations are considered, namely, a metal layer supported by a stiff plastic substrate, and a metal layer supported by a compliant elastomer substrate. The results quantitatively correlate both critical necking limit strain and necking band orientation with the material properties and thickness ratio of the substrate-metal bilayer. In particular, the predicted necking band orientation when the bilayer is under in-plane loading with a negative ratio (e.g., uniaxial tension) agrees with the slanted necking bands observed in experiments, a phenomenon that cannot be explained by existing theoretical studies assuming plane strain condition. The present study further shows that necking retardation in an elastomer-supported metal layer can allow the bilayer to absorb and dissipate more energy than an all-metal single layer with the same mass. These understandings shed light on optimal design of substrate-supported metal structures with enhanced deformability and energy absorbing capacity under complex in-plane loading conditions.

1. Introduction

Substrate-supported metal layers are being developed as structural elements and functional components in modern technologies, with the promise of enhanced mechanical performance in comparison with freestanding metal layers. For example, thin metal films deposited on polymer substrates are often used as deformable conductors and interconnects in flexible electronic devices that are often subject to large stretches, bends and twists (Cordill et al., 2010; Cotton et al., 2009; Graudejus et al., 2012; Lacour et al., 2006; Lacour et al., 2005; Li et al., 2004; Li et al., 2005b; Lu et al., 2010; Lu et al., 2007; Wagner et al., 2004; Xu et al., 2010). Polymer-coated metal layers have been shown to be able to undergo significant plastic deformation before rupture, thus hold potential as energy absorbing structural elements subject to high intensity impulsive loads (Amini

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and Nemat-Nasser, 2010; Amini et al., 2010; Morales et al., 2011; Xue et al., 2008; Xue and Hutchinson, 2007, 2008; Zhang et al., 2009).

Ductile failure of metal layers under in-plane loading often initiates from strain localization, such as the onset of necking instability (Benallal and Tvergaard, 1995; Brunet and Morestin, 2001; Franz et al., 2013; Franz et al., 2009; Haddag et al., 2009; Hashiguchi and Protasov, 2004; Needleman and Tvergaard, 1977; Neil and Agnew, 2009; Tvergaard et al., 1981; Zhang and Wang, 2012). When a metal layer is subject to modest in-plane loading, it deforms uniformly. When the loading increases to a sufficiently high level, the uniform deformation of the metal layer becomes unstable. In other words, infinitesimal perturbation of the metal layer (e.g., non-uniform thickness or pre-existing defects in the metal layer) starts to grow in amplitude, leading to decreasing thickness (i.e., local thinning or necking) in certain locations of the metal layer. On one hand, the metal layer hardens under plastic deformation (i.e., material hardening); on the other hand, local thinning leads to increased stress level at necking locations (i.e., geometric softening). When the geometric softening prevails over material hardening, the onset of necking instability in a material occurs, as attributed to Considere (1885). Localized strain in the necked region promotes the increase of stress triaxiality, which in turn causes microscopic damage and eventually leads to ductile fracture near the neck. For a substrate-supported metal layer under in-plane loading, the critical loading level for necking instability depends on the loading ratio, mechanical properties of both metal and substrate (e.g., effective incremental modulus of the metal/substrate bilayer), the metal/substrate thickness ratio, as well as the orientation of the necking band. For a given substrate-supported metal layer under a certain in-plane loading ratio, necking occurs along a certain orientation that corresponds to the lowest critical loading level. Under tension, plastics neck but the incipient strain localization often gives way to stable neck propagation along the length of the plastic layer. In other words, plastics often harden more than metals. Furthermore, many elastomers can sustain substantial stretch without suffering from necking instability, that is, these elastomers stiffen so steeply that their incremental modulus remains constant or even increases modestly upon tension. By contrast, the incremental modulus of a metal layer decreases monotonically with stretching. Consequently, under tension, a plastic/metal or elastomer/metal bilayer has a greater effective incremental modulus than a single freestanding metal layer. As a result, onset of necking instability in such substrate-supported metal layers is expected to occur at higher strains (Li et al., 2005a; Li and Suo, 2006; Xue and Hutchinson, 2007). Uniaxial tensile experiments have shown that a freestanding thin metal film usually ruptures at a small strain (Espinosa et al., 2003; Huang and Spaepen, 2000; Keller et al., 1996; Nicola et al., 2006; Pashley, 1960; Xiang et al., 2005a). By contrast, plastic-supported thin metal films can sustain tensile strains up to 50% before rupture (Alaca et al., 2002; Chiu et al., 1994; Hommel and Kraft, 2001; Lu et al., 2010; Lu et al., 2007; Macionczyk and Bruckner, 1999; Niu et al., 2007; Xiang et al., 2005b; Yu and Spaepen, 2004). It is predicted that the substrate constraint to the necking development in the metal layer disappears when the metal layer debonds from the substrate (Li et al., 2005a; Li and Suo, 2007), which has been recently verified by the experimental observation of interfacial delamination in the later stage of the tensile fracture process of a thin Cu film on a polyimide substrate (Lu et al., 2007). Necking in a single freestanding metal layer can also be retarded under dynamic stretching due to inertia effect (Guduru and Freund, 2002; Mercier et al., 2010; Mercier and Molinari, 2003; Shenoy and Freund, 1999; Sorensen and Freund, 2000; Xue et al., 2008; Zhang and Ravi-Chandar, 2006). The interaction of the substrate and inertia effects on necking retardation has also been investigated (Amini and Nemat-Nasser, 2010; Amini et al., 2010; Morales et al., 2011; Xue and Hutchinson, 2007, 2008; Zhang et al., 2009).

In practice, substrate-supported metal layers are often subject to large and complicated in-plane loading. For example, the electronic sensitive skins covering the elbow of a robot experience large biaxial stretches. The understanding of necking instability of substrate-supported metal layers under arbitrary biaxial in-plane loading, however, is poorly studied so far. Most existing theoretical studies assume plane strain condition of the bilayer deformation. Xue and Hutchinson (2007) investigated the necking retardation of elastomer-supported metal layers under biaxial loading, but the biaxial loading ratio in that study is limited to be in the positive regime. As to be shown later in the present paper, in the positive loading ratio regime, the necking band always occurs in the direction perpendicular to that of the greater tensile load. By contrast, most reported tensile experiments of substrate-supported metal layers are uniaxial tests. As commonly observed in such uniaxial tensile experiments, the incipient necking bands often occur along a slanted direction in between the two loading directions. For example, Fig. 1a shows the necking bands in a thin Cu film (170 nm thick) supported by a polyimide substrate (100 μm thick) occur along a direction about 60° away from the uniaxial loading direction. Similar experimental results showing inclined necking bands have also been reported recently (Lu et al., 2010; Macionczyk and Bruckner, 1999; Gruber et al., 2004). So far, the quantitative correlation between the necking limit strain as well as the necking band orientation and the material properties and thickness ratio of a metal/substrate bilayer in the full range of biaxial in-plane loading ratio still remains unclear.

This paper presents a comprehensive investigation to decipher the above quantitative correlation in two representative material structures, namely, a metal layer supported by a stiff plastic substrate, and a metal layer supported by a compliant elastomer substrate, respectively. In particular, bifurcation analysis predicts that a metal layer supported by a sufficiently stiff and thick elastomer substrate is immune from long wavelength necking instability. This further motivates the investigation of the enhanced energy absorption and dissipation of an elastomer-supported metal layer in comparison with that of an all-metal single layer with the same mass. It is noted that the necking limit analysis in the present study is based on a bifurcation analysis at the long-wavelength limit. In other words, such a necking instability later leads to a single localized neck in an infinitely large substrate-supported metal layer. In reality, other types of deformation instability (e.g., multiple diffusive necks or surface instability) could appear upon the onset of failure in substrate-supported metal layers, the study