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A generalized thermodynamic approach for modeling nonlinear hardening behaviors

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ABSTRACT

The capability of accurately modeling nonlinear behaviors is essential to simulation-based engineering. Giving better descriptions of actual constitutive behaviors, nonlinear kinematic hardening models are frequently considered as an ad hoc approach by directly prescribing the hardening laws. The necessity has been recognized for accommodating this effective yet empirical methodology into extreme principles which theoretically underlie the derivation of evolutionary equations in irreversible dissipative processes. In contrast to the published efforts, this paper presents a systematic approach for characterizing both nonlinear kinematic and isotropic hardening behaviors of rate-independent polycrystalline metals. With the modified principle of maximum mechanical dissipation and the method of Lagrangian multipliers, the typical rate-independent constitutive laws are derived. Enlightening decompositions of the mechanical dissipation and its implications are discussed. Control functions are introduced to provide useful specifications about formulating hardening models. In contrast to the ad hoc origins, the relationship of many existing hardening models (both nonlinear kinematic and isotropic types) has been clarified through the unified framework. Moreover both saturating and non-saturating behaviors of the two hardening types can be properly modeled and numerical implementations are presented. Particularly permanent softening can be automatically given by non-saturating kinematic hardening modeling along with other features of cyclic loading. With this approach this phenomenon is explained from the viewpoint of energy and reproduced with only one back-stress and single yield surface. Finally comparisons between the methodology in this work and other classical theories are given to clarify the relationships and analogies. Pressure-dependent yield is also discussed to show the generality of the approach.

1. Introduction

The phenomenological rate-independent plasticity theory of polycrystalline metals typically consists of the following aspects: (1) a convex yield surface; (2) the associated flow rule (i.e. the normality rule); (3) the hardening laws describing the evolution of the material's mechanical behaviors during the elasto-plastic deformation. For most polycrystalline metals, kinematic hardening and isotropic hardening are generally observed (Khan et al., 2009, 2010a,b). Kinematic hardening, accounting for phenomena under cyclic loading such as the Bauschinger effect (Liu et al., 2011), is considered to be attributed by unstable dislocations (McDowell and Moosbrugger, 1987) and often described by the second-order tensor back-stress **X** representing the translation of the yield surface (Brahme et al., 2011; Chaboche, 1986). Isotropic hardening is generally

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attributed to the accumulated dislocations (Haddadi et al., 2006) and can be described by the scalar-valued isotropic strength R characterizing the size of the yield surface (Hill, 1998).

Convexity of the yield surface and the associated flow rule are frequently proved by employing the principle of maximum plastic dissipation (Hill, 1948b; Mises, 1928). However the forms of the flow rule and the hardening rules are separately assumed in the traditional flow theory of plasticity (cf. e.g. Wu (2005)). This can be credited to the limitation that the hardening behaviors are not explicitly formulated in the classical postulate since only stress and plastic power are involved. The work by Simo and Hughes (1998) is worth noting because the hardening laws were obtained from the extreme principle along with the associated flow rule. In their methodology the principle of maximum mechanical dissipation was considered instead and the yield function was regarded as a constraint in maximizing mechanical (intrinsic) dissipation in the plastic process. The obtained hardening laws were also associated and determined by hardening potentials. Sansour et al. considered a similar approach except that the mechanical dissipation was formulated in a variational equation (Sansour et al., 2006). With this methodology, non-linear isotropic hardening can be readily obtained by selecting a proper function as the isotropic hardening potential. Yet due to the associated kinematic hardening law, only the simplest linear kinematic (LK) hardening model proposed by Prager (1956) can be derived without difficulty because the analytical form of the kinematic hardening potential is not easily determined.

Linear kinematic hardening is merely used to conceptualize kinematic hardening. Various non-linear kinematic (NLK) hardening rules have been developed to replicate the actual behaviors of materials. Armstrong and Frederick proposed the first NLK model by adding a dynamic recovery term to Prager's kinematic hardening formulation (Armstrong and Frederick, 1966). Two significant modifications of the basic Armstrong–Frederick model have been proposed: the addition of multiple back-stresses (Chaboche, 1986; Chaboche et al., 1979; Chaboche and Rousselier, 1983a,b) and incorporation of thresholds into the recovery terms (Chaboche, 1989, 1991; Ohno and Wang, 1993a,b). More NLK models have been suggested based on these approaches to improve the performances in ratcheting prediction (Abdel-Karim, 2009, 2010; Abdel-Karim and Ohno, 1998, 2000; Chen et al., 2005; Guo et al., 2011; Jiang and Sehitoglu, 1996; Kang, 2004; McDowell, 1995) and in sheet metal forming (Cao et al., 2009; Chun et al., 2002a,b; Chung et al., 2005; Geng et al., 2002; Lee et al., 2005a,b; Yoshida and Uemori, 2002). More recent researches about NLK models can be found in (Berisha et al., 2010; Li et al., 2010; Sun and Wagoner, 2011; Taherizadeh et al., 2010; Verma et al., 2011; Vladimirov et al., 2010).

Giving better descriptions of actual constitutive behaviors, NLK models are difficult to be derived from the approach by Simo and Hughes because of the challenge in determining the kinematic hardening potential. In fact these two methodologies have been put into different catalogues in some literature (Sansour et al., 2006; Zienkiewicz and Taylor, 2005). Indeed NLK models are frequently considered as an ad hoc approach that prescribes the laws of kinematic hardening without resorting to any thermodynamic extreme postulate. However these two methodologies should not be considered irrelevant to each other because both of them are proposed to describe the same plastic process. The necessity has been recognized to accommodate this effective yet empirical methodology into thermodynamic extreme principles, which theoretically underlie the derivation of various evolutionary equations in irreversible dissipative processes. Related efforts can be found about incorporating NLK models into thermodynamic orthogonality or equivalent extreme postulates (Ziegler, 1983).

Chaboche used a different plastic potential from the yield function to derive the Armstrong–Frederick model, sacrificing the consistency with the generalized normality postulate (Chaboche, 1986; Lemaitre and Chaboche, 1990). Recently this non-associated approach was also considered by Badreddine et al. (2010) and Arghavani et al. (2011) in modeling anisotropic plastic deformation in finite strain. Although this formulation retains the orthogonality rule, some common features of NLK models would be easily blurred by determining a specific potential function for each model *a posteriori*. Furthermore, although thermodynamic orthogonality leads to convexity of the plastic potential function as well as that of the domain defined by the function, convexity of the elastic domain defined by the yield function cannot be derived since these two functions are no longer identical.

Voyiadjis and Abu Al-Rub considered a similar approach to that of Simo and Hughes (1998) by employing the Lagrangian method, yet with a non-negative plastic potential F different from the yield function as a constraint (Voyiadjis and Abu Al-Rub, 2003). The Armstrong–Frederick model seems to be retained, yet this approach is mathematically vague because the Lagrangian term λF would be nonzero during the plastic process. Due to violation of the Karush–Kuhn–Tucker (KKT) optimality conditions, the objective function (i.e. the dissipation function) does not attain its maximum value for the actual state of the material (Boyd and Vandenberghe, 2004). Furthermore the relationships between different NLK models would not be readily exposed by selecting the plastic potential in a similar manner to Chaboche' approach.

Erlicher and Point suggested a parameterized pseudo-potential and derived a non-conventional loading function by Legendre–Fenchel transformation (Erlicher and Point, 2006). With their methodology the inclusion of NLK models into the class of generalized standard materials. (Halphen and Nguyen Quoc, 1975) is successfully achieved. Furthermore this approach succeeds in including the Armstrong–Frederick model and Ohno–Wang model II into a unified framework. In spite of these important results further extension of their approach would be complicated due to inadequate specifications about construction of NLK models. The formulation can be described as one of the mixed forms (see Appendix B) that are flawed by dimming the thermodynamic features about NLK models. Therefore the thermodynamic implications of some key concepts in the methodology (e.g. the decomposition of the plastic strain rate and the function giving rise to a new intrinsic time scale) could not be further revealed.

The multiplicative decomposition of the plastic deformation into energetic and dissipative parts in finite strain plasticity was proposed by Lion (2000) to account for kinematic hardening and recently considered by both (Henann and Anand, 2009)