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Multi-scale analysis in elasto-viscoplasticity coupled with damage

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ABSTRACT

The purpose of this study is to present a micromechanical approach, based on the transformation field analysis (TFA), proposed by Dvorak, which has been generalized at Onera in order to analyze the nonlinear behavior of heterogeneous materials in elasto-viscoplasticity coupled with damage. In such analysis, the macroscopic constitutive equations are not purely phenomenological but are built up from multi-scale approaches starting from the knowledge of the properties of the constituents at the microscopic or mesoscopic scales. The model can take into account some local characteristics that can evolve during the thermo-mechanical applied loads or the manufacturing process, like the grain size for metallic alloys or the fiber volume fraction for composites.

The determination of some specific tensors which are present in this formulation is closely linked to the microstructure morphology of heterogeneous materials constituting the macroscopic structure. For example, an Eshelby's based approach is more appropriate to characterize polycrystalline materials with a random microstructure, while the homogenization of periodic media technique can be used for composite materials with a sufficiently regular microstructure. The proposed methodologies allowing to perform this nonlinear analysis across the scales are illustrated with examples based on the behavior of structures reinforced with a long fiber unidirectional metal matrix composite.

1. Introduction

The increasing capabilities of experimental techniques, applied to the mechanics of materials field, allow achieving precise information from the macroscopic scale, i.e. the scale of the component, down to the atomic scale. However, pretend to analyze the macroscopic behavior of a structure starting from the molecular dynamics theory, jumping to the dynamics of dislocations (Groh et al., 2009; Gao et al., 2010) and finally calculating the plastic field on each integration point of the overall structure, is today unrealistic (McDowell, 2008, 2010). Thus, it is important to select the right information for the right purpose from the experimental data (Lissenden, 2010).

In the context of the modeling of plasticity coupled with damage, it is useful to integrate experimental observations obtained at the microscopic scale in order to propose more realistic constitutive equations. An important step is the definition of the representative volume element (RVE) (Cailletaud et al., 2003), which must be sufficiently large to be representative of the microstructure and sufficiently small not to be influenced by macroscopic mechanical gradients. For many microstructures, this RVE will be a set of sub-phases, each of them being representative of a particular geometrical or mechanical property.

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The multi-scale approach will consist to propose macroscopic constitutive equations taking into account the local behavior of each sub-phase in the RVE. The real difficulty is to establish a theoretical formalism linking macroscopic and microscopic scales when one or several sub-phases are nonlinear.

Among some multi-scale approaches proposing this link between scales, it is possible to distinguish three main categories related to:

(i) Pure analytical approaches, mostly based on the work performed by Eshelby (1957) in elasticity. Within this framework, nonlinear materials are modeled through the linearization of Eshelby's inclusion problem. This linearization can take the form of a tangent formulation (Hill, 1965), a secant formulation (Berveiller and Zaoui, 1979), an affine formulation (Masson and Zaoui, 1999; Doghri and Ouaar, 2003; Pierard et al., 2007; Doghri et al., 2011) or with a particular interaction law as proposed recently by Mercier and Molinari (2009) using the regular anisotropic tangent modulus.
(ii) Integrated approaches, in which the final goal is to describe as close as possible the real microstructure of the RVE without macroscopic constitutive equations. This category includes the multilevel finite element approach, FE^n, where n defines the number of separated scales in the analysis (Feyel and Chaboche, 2000) or the methods based on the FFT approach (Moulinec and Suquet, 1995, 2003; Lee et al., 2011). Many other theories are proposed in the literature being able to describe at the same time the local fields and the macroscopic ones. Among them, two formulations emerge, the non-uniform TFA (NTFA) proposed by Michel and Suquet (2003, 2009) and the high fidelity generalized method of cells (HFGMC) proposed by Aboudi et al. (2002).
(iii) Sequential approaches (Chaboche et al., 2001, 2005; Carrère et al., 2004) generally performed in two steps described in detail in this paper.

Section 2 presents the theoretical framework of the approach that seems the more relevant in elasto-viscoplasticity. This approach is an improvement of the method called transformation field analysis (TFA) proposed by Dvorak (1992), Dvorak and Benveniste (1992), where the RVE is decomposed in a given number of sub-phases with uniform eigenstrains inside each of them. This section discusses the accuracy of the method with respect to the more or less fine RVE discretization, which is a crucial point for this approach.

Section 3 presents the introduction of damage in the formalism as a new eigenstrain in order to avoid the calculation of some tensors each time the damage evolves and saving huge calculations when the number of sub-phases is important. This model was initially developed to describe the damage evolution at the interface of a fiber reinforced metal matrix composite. It is based on a cohesive zone model (Tvergaard, 1990) which has been generalized to a classical continuum damage model (CDM).

The new sequential approach in elasto-viscoplasticity coupled with damage based on the TFA formalism introducing the generalized eigenstrain is detailed in Section 4.

Finally, Section 5 presents one application of the fully coupled approach proposed in the previous section to analyze the nonlinear behavior of a tubular specimen loaded in torsion. The structure consists of a long fiber metal matrix composite material located between two layers of a titanium alloy. Two calculations are performed, the first one with fibers oriented at 45° inducing the highest resistance and, the second one with fibers in the axial direction of the cylinder loading the composite in its longitudinal shear direction. In this example, a post-localization technique is proposed in order to compute with the finest precision the local fields in some critical regions of the macroscopic structure.

This section will show that the macroscopic model obtained from the multi-scale approach completed by the post-localization technique (Kruch, 2007), provides a set of numerical tools used to accurately predict the nonlinear behavior of complex structures avoiding heavy finite element calculations.

2. Micromechanical analysis in elasto-viscoplasticity

Generally, a micromechanical model is based on relations (the localization relations) between the microscopic and macroscopic scales for the mechanical total strain or stress fields. In elasticity, such relations are bijections, both scales linked by the fourth order strain $\underset{\approx}{A}$ or stress $\underset{\approx}{B}$ localization tensor:

$$\begin{aligned} \underset{\sim}{\varepsilon} &= \underset{\approx}{A} : \underset{\sim}{E} \quad \text{for the strain}, \\ \underset{\sim}{\sigma} &= \underset{\approx}{B} : \underset{\sim}{\Sigma} \quad \text{for the stress}, \end{aligned} \tag{1}$$

where $\underset{\sim}{\varepsilon}, \underset{\sim}{\sigma}$ and $\underset{\sim}{E}, \underset{\sim}{\Sigma}$ are respectively the microscopic and macroscopic fields.

Tensors $\underset{\approx}{A}$ and $\underset{\approx}{B}$ generally depend on the geometry and on the mechanical properties of the microstructure. They are directly involved in the definition of the macroscopic stiffness $(\underset{\approx}{L}^{\text{hom}})$ or compliance $(\underset{\approx}{S}^{\text{hom}})$ from:

$$\begin{aligned} \underset{\approx}{L}^{\text{hom}} &= \left\langle \underset{\approx}{L} : \underset{\approx}{A} \right\rangle, \\ \underset{\approx}{S}^{\text{hom}} &= \left\langle \underset{\approx}{S} : \underset{\approx}{B} \right\rangle. \end{aligned}$$

Symbol $\langle\rangle$ represents the volume average over the RVE.