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On tapered warp-free laminates with single-ply terminations

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ABSTRACT

This article reviews the current state of the art in the design of traditional uni-directional fibre laminate construction; beyond the ubiquitous balanced and symmetric design. A ply termination algorithm is then employed to develop permissible tapered designs, with single-ply terminations and ply contiguity constraints, which are free from undesirable changes in mechanical coupling characteristics. More importantly however, is the fact that all tapered designs have immunity to thermal warping distortion; which include all combinations of anti-symmetric (or cross-symmetric), non-symmetric and symmetric angle- and cross-ply sub-sequence symmetries. Tapered designs are presented for laminates with fully uncoupled properties, and those possessing extension–shearing and/or bending–twisting coupling. Such designs represent typical fuselage skin thicknesses, i.e., with between (n =) 12 and 16 plies, but due consideration is also given to new fuselage design concepts with grid-stiffeners and/or geodesic stiffener arrangements, for which thinner designs ($n \geqslant 8$) are of interest.

1. Introduction

Tapered composite laminates have been studied extensively [1,2] in the context of delamination initiation and propagation in the region of ply terminations. However, little consideration has been given to the extent of the laminate design space and the extent to which arbitrary plies may be dropped without introducing undesirable changes in mechanical coupling characteristics or thermal warping distortion during the curing process or in-service operation.

Symmetric stacking sequences are ubiquitous in modern composite laminate design practice, for the simple reason that their use guarantees the laminate remains flat, or warp free, after high temperature curing. Non-symmetric laminates are commonly associated with, or often (incorrectly) used to describe [3], configurations that warp extensively after high temperature curing, and for which the deformed shape is also difficult to predict reliably [4]; requiring non-linear analysis techniques.

With very few exceptions, tapered designs in aircraft construction are currently certified only for balanced and symmetric laminates [5], despite the severe design constraint that 1 angle-ply termination requires a further 3 angle-ply terminations: 2 terminations to maintain balanced angle-plies, and a further 2 to maintain symmetry. Balanced stacking sequences guarantee that *Extension–Shearing* coupling is eliminated by using matching pairs of angle-ply layers [6]. Symmetric laminates guarantee that coupling between in-plane and out-of-plane behaviour is eliminated, along with thermal warping distortions that would otherwise be expected. However, tapering of symmetric laminates with fewer than the requisite 4 angle-ply terminations is believed to be common in industrial practice [7], despite the fact that the effects on the structural integrity are not well understood; these effects are simply minimized by ensuring that terminations are made at the laminate mid-plane.

An obvious, but somewhat controversial solution is to adopt unbalanced and non-symmetric stacking sequence configurations to exploit a larger, but generally unknown design space. Such laminate architectures give rise to *Extension–Shearing* and *Bending–Twisting* coupling, respectively, where *extension–shearing* coupling gives rise to *Bending–Twisting* coupling deformation in aircraft wing-box structures when top and bottom skins have identical bias fibre alignment, but this effect can be eliminated with opposing bias fibre alignment. *Bending–Twisting* coupling, at the laminate level, results in weaker compression buckling strength compared to the equivalent fully uncoupled laminate (with matching stiffness properties), although there is evidence that this continues to be ignored [7], leading to unsafe designs. *Bending–Twisting* coupling offers potential improvement in shear buckling strength with respect to the equivalent fully uncoupled laminate, but only when the resulting principal compressive stress direction and the principal bending stiffness direction are in the same sense [6].

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Recent research on laminate design has demonstrated that fully uncoupled laminates [8], or those with *Extension–Shearing* [6] and/or *Bending–Twisting* coupling [9,10] all have immunity to thermal warping distortion and, collectively, have a design space containing all possible combinations of anti-symmetric (or cross-symmetric), non-symmetric and symmetric angle- and cross-ply sub-sequence symmetries.

The results presented in this article are based on the four laminate classes, illustrated in Fig. 1 under free thermal contraction. All are immune to thermal warping distortions by virtue of the fact that their coupling stiffness properties are null ($\mathbf{B} = 0$); as would be expected from symmetric laminate configurations. Laminate classes with non-zero coupling stiffness ($\mathbf{B} \neq 0$), but with warp free or hygro-thermally curvature-stable (HTCS) properties, have been shown to require 4 ply terminations with standard ply orientations [11], but can be reduced to 2 ply terminations with non-standard ply orientations [12].

The first two classes contain balanced angle-ply layers, leading to uncoupled extensional stiffness properties. The *Simple* laminate in the first column is uncoupled in bending, whilst the laminate class in the second column possesses *Bending–Twisting* coupling. The final two laminate classes possess unbalanced angle-ply layers, leading to *Extension–Shearing* coupling properties. The laminate class in the third column is uncoupled in bending, whereas the laminate class in the fourth column has both *Extension–Shearing* and *Bending–Twisting* coupling, as would arise from unbalanced and symmetric laminates.

It should be emphasized that unbalanced laminates, otherwise referred to as *Extension–Shearing* coupled laminates, remain warp free for all solutions presented here, irrespective of the number of layers in the laminate. This is in marked contrast to other recent studies [13], where approximate solutions have been derived, which converge towards the thermo-mechanically curvature-stable, or warp-free condition only when the number of layers in the laminate becomes very large.

Similarly, symmetry has previously been shown [8,10] to be a limiting design rule, which serves only to mask the potential design space containing *Simple* and *Bending–Twisting* coupled laminates, where symmetric stacking sequences are the exception rather than the rule. Indeed, improvements in damage tolerance have been demonstrated, in the context of delamination buckling after impact [14], for fully uncoupled laminates using anti-symmetric designs; symmetric designs were found to perform no better than non-symmetric designs.

The remainder of this article is arranged as follows. Section 2 provides a summary of the derivation of definitive listings for the warp free laminate classes given in Fig. 1. Section 3 provides information on the design space for each class, including the dominant forms of sub-sequence symmetries. A ply termination algorithm is described in Section 4, which is then applied to the definitive listings of laminates from each of the four laminate classes to develop tapered laminate designs and compatible stacking sequences for single-ply terminations; for computation expedience, the definitive listings are pre-filtered against ply contiguity constraints, i.e., the maximum number of adjacent plies with the same orientation. Tapered laminate examples are presented, illustrating the limited range of symmetric solutions in comparison to other sub-sequence symmetries. Finally, lamination parameters are introduced to allow the available design space to be visually interrogated, before conclusions are drawn in Section 6.

2. Derivation of stacking sequences

The common feature relating the *Simple*, *Extension–Shearing* and/or *Bending–Twisting* coupled laminate classes is that all are decoupled, i.e. $B_{ij} = 0$; hence in-plane and out-of-plane behaviour are independent and can therefore be treated separately. The constitutive relations simplify as follows:

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \tag{1}$$

where the elements of the stiffness matrices are derived from the well know relationships:

$$A_{ij} = \sum_{k=1}^{n} Q'_{ij,\theta_k}(z_k - z_{k-1})$$

$$B_{ij} = \sum_{k=1}^{n} Q'_{ij,\theta_k}(z_k^2 - z_{k-1}^2)/2 \tag{2}$$

$$D_{ij} = \sum_{k=1}^{n} Q'_{ij,\theta_k}(z_k^3 - z_{k-1}^3)/3$$

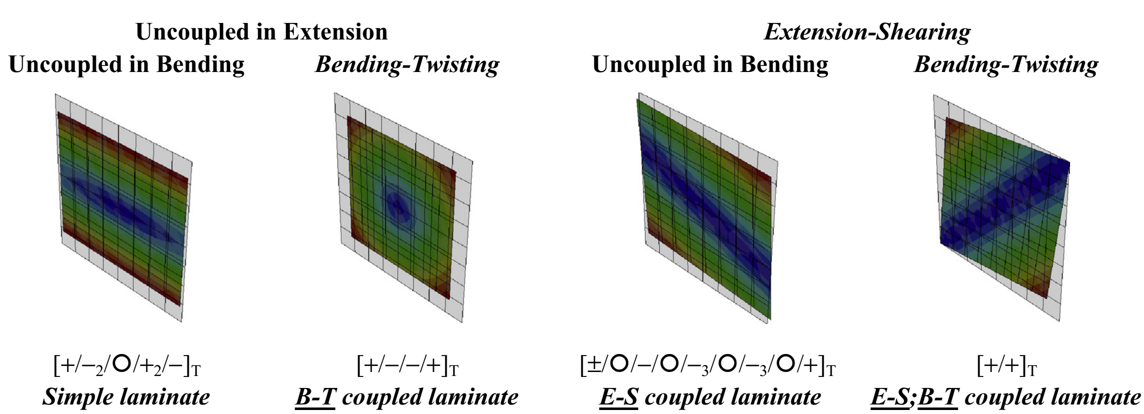

Fig. 1. In-plane thermal contraction responses (exaggerated) resulting from a typical high temperature curing process. All examples shown are square, initially flat, composite laminates. The stacking sequences provided, in symbolic form, are representative of the minimum ply number grouping for each laminate class, with standard ply orientations ±45, 0 and 90° in place of symbols +, −, ○ and ●, respectively.