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# Structural optimisation of random discontinuous fibre composites: Part 1 – Methodology

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ABSTRACT

This paper presents a finite element model to optimise the fibre architecture of components manufactured from discontinuous fibre composites. An optimality criterion method has been developed to maximise global component stiffness, by determining optimum distributions for local section thickness and preform areal mass. The model is demonstrated by optimising the bending performance of a flat plate with three holes. Results are presented from a sensitivity study to highlight the level of compromise in stiffness optimisation caused by manufacturing constraints associated with the fibre deposition method, such as the scale of component features relative to the fibre length.



## 1. Introduction

Discontinuous fibre architectures offer greater design freedom compared with conventional laminated composites, as the fibre orientation distribution, fibre volume fraction ($V_f$), and fibre length can all be locally varied in the component according to structural requirements. A number of new high volume fraction discontinuous fibre processes, such as Directed Carbon Fibre Preforming [1,2] (DCFP) and Advanced Sheet Moulding Compounds [3] (ASMC), offer exciting opportunities for structural applications within the automotive industry. Recent developments have shown that significant gains in mechanical performance can be achieved by introducing fibre alignment [4], but the current lack of robust design tools makes it difficult to exploit the versatility of these processes, limiting these materials to 'black metal' design. This often results in homogeneous isotropic fibre architectures similar to those seen in lower performance moulding compounds.

Traditional optimisation routines are primarily concerned with structural issues, such as the overall mass and stiffness of the component. Topology, shape and size are the three main categories of structural optimisation and a number of methods are well established for designing with isotropic materials, but not in the context of polymer reinforced composites. The most widely used structural optimisation methods for composite materials adopt genetic algorithms, a metaheuristic type approach [5]. These are only practical for handling discrete problems and are more widely used for optimising laminated composites where local thickness is controlled by an integer number of plies. This approach is considered to be unsuitable for optimising discontinuous fibre architectures, as the number of search points increases dramatically due to the design variables (local thickness and stiffness) being continuously variable.

Other methods, such as non-linear programming [6,7], require constant re-evaluation of the design objectives and constraints, and are therefore very computationally expensive, particularly for large structures. In comparison, optimality criterion approaches [8] use simple local rules to update design variables, which are much more efficient and suitable for complex problems. However, these approaches do not yet appear to have been adopted in the literature for optimising discontinuous fibre composites structures. CAE tools for optimising laminate structures are becoming popular, but they are still relatively immature and are unsuitable for optimising discontinuous fibre architecture, since there has been no previous demand for further development. However, as the mechanical performance of discontinuous fibre systems continue to increase, these tools will play a vital role in the wider adoption of these materials.

This paper presents a structural optimisation algorithm to adjust both local thickness and material stiffness for a DCFP component on an element by element basis. Stiffness optimality criteria is derived and the method of solving Lagrangian multipliers is adopted for each optimisation constraint, which include material volume and material cost. Solving Lagrangian multipliers is the classical approach to solving optimisation problems with equality constraints [9]. The local section thickness and stiffness values

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are updated concurrently through an iterative process, with a material cost model employed to understand the impact of increasing thickness and material stiffness during each iteration.

Structural optimisation of meso-scale discontinuous fibre architecture composites involves a combination of continuous and discrete design variables. The local thickness can be continuously varied across the component and is independent of the fibre architecture, whereas a continuous change of fibre length or tow size is impractical and therefore can only be varied in discrete regions. A segmentation algorithm is employed to ensure that the fibre architectures generated by the structural optimisation routine are suitable for manufacture [10]. Neighbouring elements with similar material properties are merged into larger zones using a common set of material parameters (fibre length and orientation, tow size etc.), controlling the local stiffness of the zone. The size and the shape of each zone are tailored to suit the fibre deposition process, so that small areas or patches with small dimensions are avoided. It is also rational that a critical minimum zone size exists in order to achieve a representative fibre architecture. The size of the representative volume element for achieving a homogeneous distribution of discontinuous fibres is known to be a function of fibre length and volume fraction [11,12].

The model has been demonstrated by optimising the bending performance of a flat plate with three large holes. The deflection and specific stiffness of the optimised DCFP panel are compared against benchmarks of a uniformly thick DCFP and steel. Sensitivity studies are also performed to illustrate the influence of key optimisation parameters on the quality of the final fibre architecture.

The modelling procedure incorporates three key areas including stiffness optimisation, material assignment and model segmentation. The optimisation process has been summarised in Fig. 1 and detailed methodology will be explained in the following sections for each key area.

## 2. Stiffness optimisation

The objective of maximising the structural stiffness is equivalent to minimising the total strain energy within the structure [8]. For an isotropic, homogeneous material under a single load case subjected to a constant volume constraint, the total strain energy is minimised when the strain energy density distribution is uniform through the part [13]. In the present work, a material such as DCFP introduces an additional design variable; the effective local modulus, therefore a new stiffness optimality criterion has been determined to optimise thickness and modulus values concurrently.

With the additional stiffness design variable, a second constraint is required to determine the limits when updating local modulus values. Restricting material cost is a sensible approach, since cost is a function of component stiffness. For example, increasing the section thickness requires a larger quantity of material to be used, whilst demanding a higher material stiffness requires an increase in fibre volume fraction or a smaller fibre tow size [14,15]. The mechanical performance for meso-scale discontinuous fibre composites is also linked to the homogeneity of the bundle ends and the number of fibre to fibre contacts [1], therefore utilising smaller, more expensive tows yields stiffer components.

The optimisation problem can be constructed as

$$\begin{aligned} &\min \quad U(E,t) \\ &\text{subject to} \quad V(t)=V_0,\ C(E,t)=C_0 \\ &\text{and} \quad E \geqslant E_{min}, t \geqslant t_{min} \end{aligned} \tag{1}$$

where $E$ and $t$ denote the modulus and thickness design variables respectively. $U$ denotes the total strain energy in the structure. $V$ and $C$ denote the overall volume and material cost of the structure, and $V_0$ and $C_0$ are the target volume and cost. $E_{min}$ is the lower bound of the modulus, which has been taken from the previous work on discontinuous carbon composites [1], and $t_{min}$ is the lower bound for thickness, selected to prevent local buckling of the structure. The minimum thickness is influenced by the lower modulus bound, since the stiffness and strength of the component changes with thickness due to the homogeneity effects [15].

The optimisation process is performed based on the results from finite element analyses of the structure. The overall strain energy, component volume and material cost can be individually expressed as a summation of the corresponding value from each finite element in the part. The optimality criterion is derived by solving the Karush–Kuhn–Tucker (KKT) conditions of the Lagrangian expression. The Lagrangian expression from Eq. (1) is

$$L = U + \lambda_1(V - V_0) + \lambda_2(C - C_0) + \lambda_3(E_{min} - E_i) + \lambda_4(t_{min} - t_i) \tag{2}$$

where $\lambda_1$, $\lambda_2$, $\lambda_3$ and $\lambda_1$ are the Lagrange multipliers corresponding to each constraint. The subscript $i$ denotes the element number. The stationary of the Lagrangian leads to the following KKT conditions

$$\frac{\partial L}{\partial E} = \sum \frac{\partial U_i}{\partial E_i} + \lambda_2 \sum \frac{\partial C_i}{\partial E_i} + \lambda_3 = 0 \tag{3}$$

$$\frac{\partial L}{\partial t} = \sum \frac{\partial U_i}{\partial t_i} + \lambda_1 \sum \frac{\partial V_i}{\partial t_i} + \lambda_2 \sum \frac{\partial C_i}{\partial t_i} + \lambda_4 = 0 \tag{4}$$

It is not possible to determine one set of Lagrangian multipliers from Eqs. (3) and (4) alone, since the number of unknowns is greater than the number of equations. However, one set of possible solutions can be calculated by choosing arbitrary values for $\lambda_3$ and $\lambda_4$, such as:

$$\lambda_3 = 0 \tag{5}$$

$$\lambda_4 = -\lambda_2 \sum \frac{\partial C_i}{\partial t_i} \tag{6}$$

Consequently Eqs. (3) and (4) can subsequently be rearranged as:

$$\frac{-\sum \frac{\partial U_i}{\partial E_i}}{\lambda_2 \sum \frac{\partial C_i}{\partial E_i}} = 1 \tag{7}$$

$$\frac{-\sum \frac{\partial U_i}{\partial t_i}}{\lambda_1 \sum \frac{\partial V_i}{\partial t_i}} = 1 \tag{8}$$

An iterative scheme for updating the element modulus can be derived by multiplying both sides of Eq. (9) by the element modulus $E$ and taking the $r$th root [16]. Similarly, an iterative scheme for updating the element thickness can be derived by multiplying both sides of Eq. (10) by the element thickness $t$ and taking the $n$th root. The recurrence relations for modulus and thickness may be written as:

$$E_i^{k+1} = \left( \frac{-\frac{\partial U_i^k}{\partial E_i^k}}{\lambda_2 \frac{\partial C_i^k}{\partial E_i^k}} \right)^{\frac{1}{r}} E_i^k \tag{9}$$

$$t_i^{k+1} = \left( \frac{-\frac{\partial U_i^k}{\partial t_i^k}}{\lambda_1 \frac{\partial V_i^k}{\partial t_i^k}} \right)^{\frac{1}{n}} t_i^k \tag{10}$$