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Compact tension specimen for orthotropic materials

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A solution for a Compact Tension (CT) specimen is proposed in order to obtain the *linear elastic fracture toughness*, the *stress intensity factor* and the *compliance at the load line*. The solution applies for any orthotropic material whose principal directions are defined by the crack direction, assuming that the crack grows along the symmetry plane of the specimen. Given two dimensionless parameters, λ and ρ, that define the orthotropy of the material, the elastic response is unique. With the aid of a parameterized Finite Element Model (FEM), a solution is obtained for any orthotropic material. The results are fitted into an interpolating function, which shows excellent agreement with simulated data. Additionally, the initial crack length required to produce a stable crack growth under displacement control is studied for various material orthotropies. Finally, some failure criteria are introduced regarding the failure at the holes of the CT and at the back end face of the specimen. Some design recommendations are given after analyzing the failure mechanisms.

1. Introduction

When Linear Elastic Fracture Mechanics (LEFM) were first developed, metallic materials were the most widely used in industry and this remains the case today. Since metals are considered to be mainly isotropic, most standardized methods [1–4] for obtaining the fracture properties, such as the critical fracture energy G_{Ic} or the stress intensity factor (SIF) K_{Ic} are developed considering isotropic materials. Despite this, most of the other materials used in industrial applications are anisotropic in nature. Woods and advanced materials, such as fiber reinforced composites, have been used increasingly in recent years. Such materials are far from satisfying the expectations of isotropy and, therefore, current standardized methods cannot be applied [5–7]. This situation means new tools and procedures need to be developed in order to measure the fracture properties of anisotropic materials.

From an LEFM point of view, and assuming that there are no inelastic energy dissipation mechanisms (except for those occurring at the crack tip), the critical fracture toughness can be seen as the elastic Energy Release Rate (ERR) per new unit area created. In the case of the CT specimen, the crack length is normalized as $\bar{a} = a/w$, where a is the crack length measured from the load application point and w is the span between the load point and the back end face of the specimen, as seen in Fig. 1. Even though the standard CT specimen has a normalized size of $w = 51$ mm, the formulation and the methodology here presented can be used for other sizes of w as long as the CT geometry is respected. With the principal directions defined as x_1 parallel to the loading direction and x_2 aligned to the symmetry plane, and assuming that the crack grows along the x_2 direction, a unique relation between the specimen compliance (C) and the normalized crack length exists. When this relation is known, it is possible to infer the crack length from the experimental compliance curve and, in conjunction with the load - load application point displacement (P_i–u_i), it is possible, ultimately, to obtain G_{Ic} or K_{Ic}. This procedure cannot be applied if the crack does not propagate along the x_2 direction, as occurs in the case of some composite materials where the majority of the plies are aligned in a direction different to x_2 [8].

Up to now, the general function $C(\bar{a})$ has only been obtained for isotropic materials [3,4]. For other types of anisotropy, current methods involve optically measuring the crack tip length during the test [7], measuring the crack tip location with the aid of the Digital Image Correlation technique [6], or the use of a Finite Element Method program [5,6,9]. Using the SIF isotropic solution on orthotropics materials can lead to significant error. For example, when computing the K_I of a T300/913 carbon epoxy cross-ply composite material with a laminate sequence of $(90,0)_{8s}$ with the standard isotropic solution, an error of 11% results with respect to that obtained by a FEM model, taking into account the orthotropy of the laminate [9]. The aim of this paper is to obtain analytical expressions of the linear elastic fracture toughness, the stress intensity factor and the compliance of the CT geometry while taking into account the orthotropy of the material. It is important to note that

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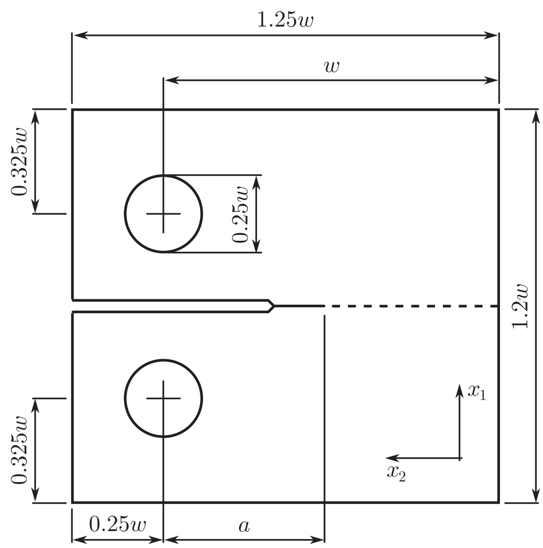

Fig. 1. Compact Tension (CT) specimen geometry, with all dimensions defined with respect to the size w, where the dashed line represents the crack path.

the same methodology described here may be used to determine the compliance and SIF functions of other specimen geometries, although the equations and results presented are only valid for the geometry of Fig. 1.

Expressions such as the ones presented here are useful to obtain G_{Ic} or K_{Ic} from experimental results. As previously stated, they can be found in many Fracture Mechanics handbooks [3,4] and in standardized procedures [1,2] only for the isotropic case. Obtaining the expressions for the orthotropic case may improve the level of standardization on how to obtain fracture properties of non-isotropic materials, instead of computing a particular FEM solution for every different material that needs to be tested. Also, in some cases, an explicit expression of the SIF in terms of the crack length is needed when solving more complicated Fracture Mechanics problems, such as crack-bridging models and cohesive models. A linear FEM is not enough to solve this type of problems, and non-linear constitutive models are needed, which require high computational time and resources. The use of explicit expressions, like the ones presented in this paper, can help to reduce the computing times drastically.

This paper is structured as follows: Section 2 defines the stress field of a planar orthotropic solid as a function of two dimensionless parameters, λ and ρ, which define the orthotropy of the material. Section 3 presents the procedure and assumptions of the FEM models. Section 4 contains a parametric function of the compliance and the SIF for a wide range of orthotropies. In Section 5, the stability of the crack growth is studied from a linear elastic point of view. Section 6 presents some design recommendations based on the proposed material failure criteria. Finally, Section 7 summarizes the conclusions and describes the relevance of this work.

2. Stress field of a planar orthotropic solid

In a bi-dimensional problem defined by the x_1–x_2 plane, the stress state of an elastic body with its boundary conditions (BCs) prescribed only by tractions depends solely on the BCs, the geometry and two dimensionless parameters that define the anisotropy of the material [10]. Consequently, for any given isotropic material, these values remain constant and, therefore, the stress state does not depend on the material. This property of the stress state means it is relatively simple to generate $C(\bar{a})$ and stress intensity factor curves.

Given a general anisotropic material with a linear constitutive relation, in a bi-dimensional problem, the stress–strain relation can be expressed as:

$$\varepsilon_i = \sum_{j=1,2,6} b_{ij}\sigma_j, \quad i = 1,2,6 \tag{1}$$

where:

$$b_{ij} = \begin{cases} s_{ij}, \text{for plane stress} \\ s_{ij} - s_{i3}s_{j3}/s_{33}, \text{for plane strain} \end{cases} \quad i,j = 1,2,6. \tag{2}$$

It is known that for any anisotropic material, the solution of the differential equation that defines the stress state depends on the roots of the characteristic polynomial [10]:

$$b_{11}p^4 - 2b_{16}p^3 + (2b_{12} + b_{66})p^2 - 2b_{26}p + b_{22} = 0 \tag{3}$$

with four complex roots in p. If the material is orthotropic with the principal directions x_1–x_2 defined by the principal axes of the material, only four independent elastic constants are needed: $b_{11},\ b_{12} = b_{21},\ b_{22}$ and b_{66}, since $b_{16} = b_{26} = 0$. Hence, Eq. (3) is reduced to:

$$\lambda p^4 + 2\rho\sqrt{\lambda}p^2 + 1 = 0 \tag{4}$$

where p_1 and p_2 are the roots with positive imaginary parts and:

$$\lambda = \frac{b_{11}}{b_{22}}, \quad \rho = \frac{2b_{12} + b_{66}}{2\sqrt{b_{11}b_{22}}} \tag{5}$$

In the plane stress case, λ and ρ are expressed as:

$$\lambda = \frac{E_{22}}{E_{11}}, \quad \rho = \frac{\sqrt{\lambda}}{2G_{12}}(E_{11} - 2\nu_{12}G_{12}) \tag{6}$$

where E_{11} and E_{22} are the elastic moduli, G_{12} is the shear modulus, and ν_{12} is the Poisson's ratio. In the plane strain case, λ and ρ are obtained by replacing $E_{11},\ E_{22}$ and ν_{12} in Eq. (6) by:

$$E'_{11} = \frac{E_{11}}{1 - \nu_{13}\nu_{31}}, \quad E'_{22} = \frac{E_{22}}{1 - \nu_{23}\nu_{32}}, \quad \nu'_{12} = \frac{\nu_{12} + \nu_{13}\nu_{32}}{1 - \nu_{13}\nu_{31}} \tag{7}$$

To ensure the positive definiteness of the strain energy, it must be ensured that:

$$\lambda > 0 \quad \text{and} \quad \rho > -1 \tag{8}$$

The anisotropy of the material is easily described by the parameters λ and ρ. For an isotropic material, the parameters take the values $\lambda = \rho = 1$. However, for a cubic material, it only needs to be ensured that $\lambda = 1$ and that $\rho \neq 1$. Table 1 contains the values of λ and ρ for a number of materials. From the point of view of composite laminates, the laminate anisotropy is determined by the lay-up sequence; an in-plane isotropic lay-up may have its principals axes oriented in any direction by definition. Some examples of laminate sequences that satisfy this condition are $[0,\pm60]_s$, $[0,\pm45,90]_s$ or $[0,\pm36,\pm72]_s$. On the other hand, cubic materials have a principal axis every 45°. An example of cubic laminate sequence is a cross-ply laminate.

Table 1
Values of λ and ρ for seven different materials.

Material	λ	ρ
T300/920 unidirectional lamina [19]	0.0657	3.7326
T300/920 $[0,\pm60]_s$ isotropic	1.0	1.0
T300/920 $[0,90]_s$ cubic	1.0	7.9302
Western White Pine wood [20]	0.0380	1.9635
Northern White Cedar wood [20]	0.0810	0.6642
Cu (*FCC*) [21]	1.0	0.03
Fe (*BCC*) [21]	1.0	0.20