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A new semi-empirical law for variable stress-ratio and mixed-mode fatigue delamination growth

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ABSTRACT

A new semi-empirical equation that describes the fatigue delamination growth in fibre reinforced toughened epoxies is presented and validated against data available in the literature. The new law accounts for the simultaneous effects of the stress-ratio and mode-mixity on the interlaminar crack propagation. If delamination propagation thresholds are ignored, the proposed semi-empirical equation allows describing interlaminar crack propagation employing only three material dependent parameters, whereas alternative models presented in the literature require four. If reliable threshold data are available from experimental tests, the new semi-empirical law can be extended to a unified description of stress-ratio, mode-mixity and thresholds effects using six material dependent parameters.

1. Introduction

Delaminations are recognised as the main source of structural failures in composite structural elements. The process of fatigue damage accumulation in fibre reinforced plastics starts with the onset of matrix cracking in off-axis plies, which then leads to the initiation of delaminations at interfaces. The energy release rate (ERR) at the delamination tip has been identified as the governing variable for delamination onset and growth in a fatigue regime [1].

It has been observed that the fatigue delamination growth (FDG) rate in fibre reinforced materials can be expressed as a function of the interlaminar crack tip ERR via a two parameter semi-empirical power law [2], that comprises a pre-factor and an exponent. Generally speaking this represents an extension of the classical Paris–Erdogan equation originally formulated for alloys [3], when the relation holding between the stress intensity factor (SIF) and ERR for anisotropic materials is taken into account [4]. The original Paris–Erdogan law was formulated in terms of the SIF range, whilst semi-empirical FDG equations are given either in terms of ERR peak or range. However the latter choice leads to difficulties since the ERR range decreases for negative stress ratios and it is actually zero in fully reversed loading. This implies that the crack driving force should decrease at negative stress-ratios, whilst the experimental evidence [26] clearly shows that load reversals accelerate the delamination growth rate. In order to overcome these issues, Schön [5] proposed to consider the "range of change" in ERR as the physical quantity governing fatigue delamination growth for negative stress-ratios. This implies that, in fully reversed loading, the delamination growth rate depends on twice the peak ERR attained during a fatigue cycle.

Gustafson and Hojo [6] introduced a semi-empirical FDG power law for carbon/epoxy laminates based on the ERR range and observed that the pre-factor and exponent are dependent on the material and mode-mixity associated with the loading regime. The latter is here defined as the ratio between the mode II ERR and the total ERR. They proposed that the FDG rate in mixed-mode conditions can be expressed as an average of those attained in pure mode I and pure mode II conditions. The weight factor in such an average is given by the mode-mixity itself. They also observed that the stress-ratio has a considerable effect on FDG, especially in mixed-mode conditions.

Russell and Street [7] performed compressive mixed-mode fatigue tests on delaminated sandwich panels and expressed the FDG via a power law as a function of the ERR range normalised by the static fracture toughness. They again considered the mixed-mode FDG to be an average of those characterising mode I and mode II propagation.

Dahlen and Springer [8] introduced a mixed-mode FDG power law derived from dimensional analysis. The proposed expression comprises a peak ERR term normalised by the material fracture toughness and it also includes an effective stress-ratio term which allows accounting for shear reversals in mode II, based on Elber's original crack closure concept. The effect of mode-mixity is accounted for via weight factors that comprise both the peak ERR and the ratio of static fracture toughness values for pure mode I and pure mode II.

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Kardomateas et al. [9] introduced a mixed-mode FDG equation where the ERR range is normalised with respect to the static fracture toughness. They assumed that the fatigue pre-factor and exponent have the same dependency on the mode-mixity as that characterising the material toughness according to a linear interaction fracture criterion.

Kenane and Benzeggagh [10] employed a power law to describe the mixed-mode FDG as a function of the ERR range in glass–epoxy composites. They observed that the pre-factor is a decreasing function of the mode-mixity, while the power law exponent increases. They also provided semi-empirical expressions in order to describe the aforementioned influence of the mode-mixity on the fatigue pre-factor and exponent.

Andersons et al. [11] developed a FDG model which is based on the application of a linear damage accumulation rule under the assumption that the stress field at the delamination tip can be regularized. They assumed the material failure at the interlaminar crack front is described by a quadratic interaction criterion and they consequently derived a mixed-mode FDG power law where the independent variable is the sum of the peak mode I and mode II ERR, both normalised by their respective static toughness values. They also observed that the stress-ratio strongly influences the FDG rates and such effect can be described by a rotation of the FDG rate versus peak ERR curves, whose slope increases as the stress-ratio approaches the value of one.

Assuming that the static strength and toughness properties of the fibre reinforced material are known, the semi-empirical FDG models in Refs. [6–8,10,11] require the introduction of four independent fatigue related material parameters, which have to be identified by experimental tests. The FDG law presented in Ref. [9] requires only two independent material parameters to describe the fatigue behaviour, but it has been demonstrated [11] that it may lead to a substantial underestimation of the interlaminar crack propagation rates. Moreover the effect of stress-ratio on the FDG is not accounted for in all the models mentioned above, with the exception of the semi-empirical law proposed by Andersons et al. [11].

In this paper the emphasis is placed upon providing a general data reduction tool which can be employed to predict the FDG rates in the engineering design of damage tolerant composite structures. Therefore a new semi-empirical FDG equation is presented and validated with the help of experimental data available in the literature. The proposed new FDG law allows describing the effects of mode-mixity and stress-ratios on FDG in fibre reinforced toughened epoxies. The new semi-empirical FDG equation is valid only for stress-ratios values that are positive and smaller than one. Thus the effect of load reversals is not included in the FDG model proposed here and this aspect of fatigue delamination growth will be addressed in a future paper. If propagation thresholds are neglected, the new FDG equation requires introducing only three independent fatigue related parameters. It will be also demonstrated that it is possible to extend the aforementioned equation to the description of near-threshold growth regimes by adding a further three material parameters, namely threshold SIF ranges and a threshold sensitivity factor. It is worth observing that, to the authors' knowledge, none of the other FDG models available in the literature allows the simultaneous description of positive stress-ratio, mode-mixity and threshold effects.

2. Semi-empirical fatigue delamination growth law

2.1. Governing equation

The following semi-empirical mixed-mode FDG law is here postulated for proportional loading

$$\frac{da}{dN} = C\left[\frac{G_{\max}}{G_C(\phi)}\right]^{\frac{b_{0I}}{(1-R)^{1+\alpha(\phi)}}e^{-h\phi}} \tag{1}$$

where $G_{\max}$ is the peak mixed-mode ERR, $G_C(\phi)$ is the mixed-mode fracture toughness, $R \in [0,1)$ is the stress-ratio and $\phi \in [0,1]$ the mode-mixity. Regarding the aforementioned variables, the definitions below hold

$$G_{\max} = G_{I\max} + G_{II\max}R = \sqrt{\frac{G_{\min}}{G_{\max}}}\phi = \frac{G_{II\max}}{G_{\max}} \tag{2}$$

where $G_{I\max}$ and $G_{II\max}$ are respectively the mode I and mode II peak ERR. According to the definition given in the last of Eq. (2), the mode-mixity is zero in pure mode I and equal to one in pure mode II. Therefore here an increasing mode-mixity ϕ indicates the progressive transition from delamination opening, i.e. mode I, to pure sliding, i.e. mode II.

In Eq. (1) the material dependent parameters that govern the delamination propagation are the pre-factor C, the exponent b_{0I} and the mode-mixity coefficient h. The exponent b_{0I} gives the slope of the da/dN versus $G_{\max}/G_C$ curve in pure mode I and for a stress-ratio $R = 0$.

Finally $\alpha(\phi)$ is a function of the material mixed-mode fracture toughness $G_C(\phi)$

$$\alpha(\phi) = \frac{G_C(\phi) - G_{IC}}{G_{IIC} - G_{IC}} \tag{3}$$

where G_{IC} and G_{IIC} are respectively the mode I and mode II fracture toughness values. A justification of the expression here postulated for the function $\alpha(\phi)$ is given in Section 2.4. The $G_C(\phi)$ in Eqs. (1) and (3) may be given using the expression derived from a linear interaction criterion for fracture [12]

$$G_C^{(Li)}(\phi) = \frac{G_{IC}G_{IIC}}{G_{IC}\phi + G_{IIC}(1-\phi)} \tag{4}$$

Alternatively the mixed-mode fracture toughness can be expressed using the equation proposed by Benzeggagh and Kenane [13]

$$G_C^{(BK)}(\phi) = G_{IC} + (G_{IIC} - G_{IC})\phi^n \tag{5}$$

Eq. (1) describes the FDG as a power law of the normalised peak ERR, similarly to what assumed in Refs. [8,11]. The main difference with the other FDG models presented in the literature is that in Eq. (1) the effects of the mode-mixity and stress-ratio are entirely accounted for by the exponent to which the normalised ERR is raised.

2.2. Remarks on the stress-ratio effect

The effect of an increase of the stress-ratio on the FDG rate expressed as a function of the peak ERR is to slow down the delamination propagation while making the slope of the da/dN versus $G_{\max}/G_C$ curve steeper. This has been reported in the literature for both mode I [14–16] and mode II [17–19] FDG in fibre reinforced toughened epoxies. Thus the same effect is here assumed to occur also in a mixed-mode regime. This dependence of the FDG rate on the stress-ratio is easily explained by observing that increasing R in the [0,1] range reduces the stress amplitude, which controls the delamination propagation in toughened epoxies [14]. As a supporting evidence of this explanation it was also observed in the literature that at a fixed mode-mixity the stress-ratio does not affect the fracture surface morphologies [15]. Those are the same in fatigue as in static loading both in mode I and in mode II, so the stress-ratio alone cannot influence the mechanism governing the FDG at the micro-scale.

The semi-empirical FDG law postulated in Eq. (1) predicts that an increasing stress-ratio at constant mode-mixity produces an