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Comprehensive analytic solutions for the geometry of symmetric constant-wall-thickness scroll machines

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ABSTRACT

In this work, a comprehensive framework and geometric solutions are presented for the most common type of scroll machine, the symmetric constant-wall-thickness scroll compressor based on the involute of a circle. This analytic treatment comprises solutions for the volume, derivative of volume, centroid, normalized force and moment parameters, and leakage areas for all chambers in the machine. The representations for the centroid and area are based on Green's Theorem analysis which allows for general solutions that can then also be applied to other machines. Analytic solutions are also presented for the discharge geometry corresponding to arc-line-arc, perfect-meshing-profile, one-arc, and two-arc discharge geometries. Each of the derived parameters is validated against numerical solutions based on polygons. Source code in the Python programming language is provided for the analytic derivations as well as the implementation of each of the analytic outputs.

Solutions analytiques générales pour la géométrie des machines à spirale à épaisseur de paroi constante

Mots clés : Compresseur à spirale ; Détendeur à spirale ; Géométrie

1. Introduction

The scroll machine was first proposed by Léon Creux (1905) in 1905, but it would be many decades before scroll compressors would see practical application due to the manufacturing challenges posed by the involutes.

A significant amount of literature exists which deals with the geometric modeling of scroll compressors. Morishita and Sugihara (1984) produced one of the first complete

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Nomenclature

Variables

A	Area (m^2)
$\tilde{A}$	Area contribution (m^2)
b_l	Intercept of line
d	Distance (m)
f	Anti-derivative for force
$\mathbf{F}$	Force vector (N)
h	Height of scroll (m)
Gr	Anti-derivative for area (m^2)
$\hat{i}$	Unit vector in $+x$ direction
$\hat{j}$	Unit vector in $+y$ direction
L	Length (m)
L	Green's theorem function
M	Green's theorem function
m_l	Slope of line
$\mathbf{M}$	Moment (N m)
$\mathbf{n}$	Unit normal vector
N_{c1}	Number of c_1 chambers
N_{c2}	Number of c_2 chambers
$N_{c,max}$	Number of pairs of compression chambers
p	Pressure (Pa)
r	Radius (m)
t	Parameter for parameterization
t_s	Scroll thickness (m)
u	Solution value for ϕ_{ssa} (rad)
Δx	Difference in x coordinate (m)
Δy	Difference in y coordinate (m)
x	Cartesian coordinate (m)
$\bar{x}$	Centroid coordinate (m)
y	Cartesian coordinate (m)
$\bar{y}$	Centroid coordinate (m)
α	Angle in discharge curve calculation (rad)
α	Index of compression chamber pair
β	Angle in discharge curve calculation (rad)
δ	Solution value for ϕ_{ssa} (rad)
ϕ	Involute angle (rad)
θ	Crank angle (rad)
θ_d	Discharge angle (rad)
Θ	Shifted orbiting angle ($\Theta = \phi_{fie} - \theta - \pi/2$) (rad)
ϖ_a	Parameter for $r_{a2,max}$
V	Volume (m^3)
V_{ratio}	Built-in volume ratio
δ_{radial}	Radial gap width (m)

Superscripts

/	Line segment
◡	Arc of circle
⊃	Involute

Subscripts

a	Axial
$a1$	Discharge arc 1
$a1,1$	Discharge arc 1 point 1
$a1,2$	Discharge arc 1 point 2
$a2$	Discharge arc 2
$a2,1$	Discharge arc 2 point 1
$a2,2$	Discharge arc 2 point 2
$a2,max$	Maximum for discharge arc 2
b	Base (generating)
disp	Displacement
D	Downstream
o	Orbiting
0	Initial
s	Starting
e	Ending
f	Fixed scroll
$f{-}o$	From fixed to orbiting scroll
is	Inner involute starting
os	Outer involute starting
ssa	Suction separation
$fi0$	Fixed scroll inner involute initial
fis	Fixed scroll inner involute starting
fie	Fixed scroll inner involute ending
$fo0$	Fixed scroll outer involute initial
fos	Fixed scroll outer involute starting
foe	Fixed scroll outer involute ending
I	Inner for chamber
k,fi	Conj. pts. fixed scroll inner involute
k,fo	Conj. pts. fixed scroll outer involute
k,oi	Conj. pts. orb. scroll inner involute
k,oo	Conj. pts. orb. scroll outer involute
max	Maximum
min	Minimum
om	Overturning moment
o1	Arc 1 on orbiting scroll
o2	Arc 2 on orbiting scroll
$o{-}f$	From orbiting to fixed scroll
oi0	Orbit. scroll inner involute initial
ois	Orbit. scroll inner involute starting
oie	Orbit. scroll inner involute ending
oo0	Orbit. scroll outer involute initial
oos	Orbit. scroll outer involute starting
ooe	Orbit. scroll outer involute ending
o	Orbiting scroll
O	Outer for chamber
r	Radial
t	Tangential
U	Upstream
x	x-component
y	y-component
○	Spinning

analyses of the scroll compressor including geometry and dynamics. Yanagisawa et al. (1990) constructed a full geometric model based on the foundation set by Morishita. More treatment of the scroll geometry is available from Halm (1997). One of the major shortcomings of Halm's model is that the derivation is based on particular values for the initial angles of involute. Wang et al. (2005) noted this problem and carried out derivations for the scroll chamber