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Analytical solution for decelerated mechanochemical corrosion of pressurized elastic–perfectly plastic thick-walled spheres

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ABSTRACT

The paper presents benchmark solutions for the mechanochemical corrosion of an elastic–perfectly plastic thick-walled spherical shell under internal and external pressure with possible inhibition of corrosion being taken into account. The rates of inner and outer corrosion are linear with equivalent tensile stress at the related surface when the stress increases beyond a certain threshold. The time of initial and complete yielding is determined and compared with the same results for a cylindrical tube that were obtained earlier.

1. Introduction

Doubly curved thick-walled shells are often used in metal structures, including pressure vessels, automobile bodies, ship hulls, pipes, liquid storage tanks, etc. A hollow thick sphere is still one of the classical models in continuum mechanics, for which analytical benchmarks can be obtained in many cases. Most metal structures are exposed to corrosive environments in addition to being subjected to mechanical loads. Such operating conditions may activate the process of mechanochemical wear, which is more severe than the simple superposition of destructions caused by stresses and chemical reactions acting separately [1–5]. In some cases, the corrosion rate can grow almost linearly with the stress when the load exceeds a certain threshold [1,2,6,7]. Corrosion can be concentrated locally, or it can extend across a wide area nearly uniformly. It is worth noting that the term "mechanochemical corrosion" was introduced by E.M. Gutman with regard to general corrosion facilitated by stress [3,4]. Corrosion may often be considered as uniform in the case of elastic deformation. In plastic region, significant electrochemical heterogeneity of the surface may be developed and therefore the term "mechanochemical corrosion" is not always applicable. General wear can occur both under the formation of a fully protective layer of corrosion products, and in the absence of oxide or biofilms. The formation of a closed protective coating, as well as the change in concentration of one or other reactants, can show inhibiting effects [2,8,9], when the corrosion rate can be supposed to follow an exponential decay with time.

When corrosion rates depend on stresses, and stresses, in turn, depend on decreasing (due to corrosion) cross-sectional dimensions of an element, one has to solve an initial boundary value problem with unknown boundaries. Such problems are mostly studied by numerical methods. Analytical solutions have been found for the cases when corrosion rates are expressed only through one parameter changing with time (*e.g.* thickness or mean stress) for a uniform mechanochemical wear. Closed-formed expressions were derived in [10,11] for loaded cylindrical and spherical shells under the assumption of exponential dependence of anodic dissolution rates on the average normal stress. Some exact solutions using linear relationships between corrosion rate and equivalent stress were obtained by the authors of [6,7,12–14]. Most of these studies are devoted to bars, plates and thin-walled shells [6,11–13]. More complex tasks of the mechanochemical corrosion of thick-walled tubes were solved analytically in [15–17]. The numerical modelling of idealized elastic–plastic pipes with corrosion pits in the wall was given in [18].

The paper presented here is concerned with the mechanochemical corrosion of an elastic–perfectly plastic thick sphere under external and internal pressure. A piecewise linear relationship is used between the corrosion rate and the equivalent tensile stress: the rates of inner and outer corrosion are linear with stresses at the related surface when the stresses increase beyond certain thresholds. The possible inhibition of corrosion is also taken into account. Particular cases of this problem were considered in [19,20]. Herein, the complete analysis of the problem is performed. In Section 3,

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analytical solutions are obtained for the corrosion process during the pure elastic deformation regime for both cases: when the equivalent stress is less or greater than the threshold value. In Section 4, the corrosion of the sphere during the partially plastic stage is discussed. The duration of plastic-zone propagation throughout the sphere's wall is determined here. For visual assessment of the time of initial and complete yielding of the shell, some estimating functions are introduced in Section 5. Several examples demonstrate the effect of the parameters of the metal–environment system on the sphere's lifetime. The results are compared with the same ones for the elastic–perfectly plastic cylinder. All the presented solutions are useful for design purposes and as a benchmark for numerical analyses.

2. Problem formulation

Consider a hollow thick sphere, with the inner radius r and the outer radius R subjected to internal pressure p_r and external pressure p_R of chemically reactive media. For the sake of generality, in Section 3.1, the solution for the solid sphere ($r = 0$) under pressure is also described. The inner and outer radii of the sphere at the initial time $t = 0$ are denoted by r_0 and R_0, respectively,

$$R(0) = R_0, \quad r(0) = r_0. \tag{1}$$

From the moment $t = 0$, the shell is assumed to dissolve uniformly and its inner and outer radii change with time t. The rates of corrosion at the internal and external surfaces are given by the formulas [2,6,7]:

$$v_r = \frac{dr}{dt} = v_r^0 \exp(-bt) \quad \text{at} \quad \sigma_i(r) \leqslant \sigma_r^{th}, \tag{2}$$

$$v_r = \frac{dr}{dt} = [a_r + m_r\sigma_i(r)] \exp(-bt) \quad \text{at} \quad \sigma_i(r) \geqslant \sigma_r^{th}, \tag{3}$$

$$v_R = -\frac{dR}{dt} = v_R^0 \exp(-bt) \quad \text{at} \quad \sigma_i(R) \leqslant \sigma_R^{th}, \tag{4}$$

$$v_R = -\frac{dR}{dt} = [a_R + m_R\sigma_i(R)] \exp(-bt) \quad \text{at} \quad \sigma_i(R) \geqslant \sigma_R^{th}. \tag{5}$$

Here, m_r, m_R are the observable proportionality coefficients between the rates of dissolution and the stresses at the surface involved; b is a decay constant (index of corrosion inhibition); $a_r = v_r^0 - m_r\sigma_r^{th}$ and $a_R = v_R^0 - m_R\sigma_R^{th}$; σ_r^{th} and σ_R^{th} are threshold stresses for inner and outer corrosion, correspondingly; v_r^0 and v_R^0 are initial corrosion rates at $t = 0$ for $\sigma_i(r) < \sigma_r^{th}$ and $\sigma_i(R) < \sigma_R^{th}$, respectively; $\sigma_i = \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}/\sqrt{2}$ is the equivalent tensile stress; σ_1, σ_2, and σ_3 are principal stresses. The constants a_r, a_R, m_r, and m_R are often different for tension and compression despite the fact that σ_i is always positive [2]. The chemistry of solid–media interaction is supposed to be lumped into the above kinetic parameters. The thresholds σ_r^{th} and σ_R^{th} are mostly independent of mechanical characteristics of the shell's material, such as ultimate stress and yield point, and depend on the properties of the metal–media system [2]. For some neutral, alkalescent or weakly acidic environments, the threshold can be equal to $\sigma^{th} = \infty$. It is also to be noted that, while for $b > 0$, multiplier exp $(-bt)$ is meant to describe a process of inhibition of corrosion [2], for $b < 0$, this multiplier can be used to describe corrosion acceleration if $m_r = m_R = 0$ [7]. If exponents b are different for internal and external corrosion, the problem can only be solved numerically. Since we focus on analytic transformations, these situations are not considered here.

The sphere's material is modelled as an idealized elastic–plastic material with a well-defined yield strength, σ_y. According to the von Mises yield criterion, the yielding of material is assumed to begin when its von Mises stress exceeds the yield strength: $\sigma_i = \sigma_y$.

The problem is how to determine the equivalent stress at any time t and to assess the shell lifetime (without an allowance for buckling). The support conditions are not taken into account. Moreover, it is of interest to find out if the time of plastic zone propagation throughout the sphere gives an essential addition for its lifetime and to compare these results with the same data for the thick-walled cylinder.

3. Problem solution for the pure elastic stage

Let the equivalent stress σ_i through the body at $t = 0$ be below the yield stress, σ_y. Let $t^e > 0$ be the time of the end of the perfectly elastic stage, when σ_i at some point reaches the yield limit. Then the sphere's material remains entirely elastic until the instant t^e. With reference to the spherical coordinates ρ, θ, φ (with the origin at the centre of the sphere), the stress-components at this stage are determined by the well-known solution of the Lame problem for a pressurized thick-walled spherical shell. From the spherical symmetry of the problem, it follows that the radial and both tangential directions are principal axes of stresses: $\sigma_1 = \sigma_2 = \sigma_{\theta\theta} = \sigma_{\varphi\varphi}$ and $\sigma_3 = \sigma_{\rho\rho}$. Therefore, $\sigma_i(\rho)$ can be written in the form:

$$\sigma_i(\rho) = |\sigma_{\theta\theta}(\rho) - \sigma_{\rho\rho}(\rho)|. \tag{6}$$

Substituting the Lame's solution into Eq. (6), the equivalent stress at the inner and outer surfaces at any $t \in [0, t^e]$ can be found as:

$$\sigma_i(r) = \frac{3\Delta p}{2} \frac{\eta^3}{\eta^3 - 1}, \tag{7}$$

$$\sigma_i(R) = \frac{3\Delta p}{2} \frac{1}{\eta^3 - 1}, \tag{8}$$

where $\Delta p = |p_r - p_R|$ and

$$\eta = R/r. \tag{9}$$

The equivalent stress is at the maximum at the inner boundary:

$$\sigma_i(r) = \max_{r \leqslant \rho \leqslant R} \sigma_i(\rho).$$

From this, it follows that yielding begins at the bore of the sphere. So it is reasonable to trace $\sigma_i(r)$ to determine the time t^e of the end of the pure elastic period when $\sigma_i(r)$ just reaches σ_y. Let $\sigma_i(r)$ be denoted by σ. From Eq. (7), using this denotation, the ratio η can be expressed as:

$$\eta = \sqrt[3]{\frac{\sigma}{\sigma - 3\Delta p/2}}. \tag{10}$$

Initial conditions to be satisfied at $t = 0$ are (1) or

$$\sigma^0 = \sigma_i(r)\big|_{t=0} = \frac{3\Delta p}{2} \frac{\eta_0^3}{\eta_0^3 - 1}, \quad \eta_0 = \frac{R_0}{r_0}. \tag{11}$$

3.1. The case of homogeneous stress

When $\{r = 0, p_R = p\}$ or $\{r \neq 0, p_r = p_R = p\}$, homogeneous stress irrespective of the corrosion process occurs in the sphere: $\sigma_{\rho\rho} \equiv \sigma_{\theta\theta} \equiv \sigma_{\varphi\varphi} \equiv -p$, then $\sigma_i = 0$, and Eqs. (2) and (4) hold. Thus, the von Mises stress σ_i never reaches the yield limit up to the time at which thickness of the shell, $h = R - r$, becomes equal to zero. In such cases, lifetime is defined as the time to complete dissolution. Let us denote this time by t^d. Integrating Eqs. (2) and (4) with initial conditions (1) gives:

$$r = r_0 + v_r^0[1 - \exp(-bt)]/b \quad \text{at} \quad b \neq 0, \tag{12}$$

$$r = r_0 + v_r^0 t \quad \text{at} \quad b = 0, \tag{13}$$

$$R = R_0 - v_R^0[1 - \exp(-bt)]/b \quad \text{at} \quad b \neq 0, \tag{14}$$

$$R = R_0 - v_R^0 t \quad \text{at} \quad b = 0. \tag{15}$$