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Dynamic analysis of rectangular plates with a single side crack and in contact with water on one side based on the Rayleigh–Ritz method

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ABSTRACT

A computational method is proposed based on the Rayleigh–Ritz method for the analysis of dynamic characteristics of a baffled rectangular plate with an arbitrary side crack and in contact with an infinite water domain on one side. Based on the hypothesis that the water is incompressible, inviscid and the motion is irrotational, the added mass density expressed by the displacement trial functions is obtained by means of Green function method. The behavior of crack is described by the corner functions, and the natural frequencies and the corresponding modes are calculated through the Rayleigh–Ritz technique. The validity and the accuracy of the proposed method are demonstrated based on the comparison with the results obtained by ANSYS and those published in other literature, and the influences of water on the natural frequencies and mode shapes of the rectangular plates with side cracks are investigated. It is illustrated that the differences between dry and wet mode shapes increase with increasing mode order on the whole, except for the completely free plates, and the differences are evident for some higher modes.

1. Introduction

Fluid–structure interaction (FSI) problems can be found in many engineering fields, such as floodgates, liquid containers, pontoon bridges, sea drilling platforms, the local vibrations of submarines and ships, etc. It is well known that in the presence of liquid, the natural frequencies of the structures decrease compared with those in vacuums. Therefore, an important aspect of this kind of problems, which has received much attention, is determining the dynamic characteristics of the structures. Moreover, due to the corrosion, fatigue and other reasons, the dynamic characteristics of structures will also be altered because of the appearance of crack (Gummer and Hensman, 1992; Huang and Kuang, 2002; Marín et al., 2009). The dynamic characteristics of structures are closely related with the design, health monitoring and safe operation of equipments. Therefore, the prediction of their changes due to the FSI or crack is of great importance. In view of the wide application of plate in engineering fields, comprehensive research has been conducted to investigate the influences of the liquid or crack on the dynamic characteristics of the plates, respectively.

For the vibration with FSI, various methods have been developed to solve this kind of problem. Nagaya and Takeuchi (1984) solved vibration problems of arbitrary shaped plates in contact with fluids, and the influence of water was treated as pressure, which is obtained based on Navier–Stokes' equation and coupling condition. Montero De Espinosa and

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Gallego-Juarez (1984) developed a simple approximate method to study the influence of water on the natural frequencies of the circular plates whose dry mode deflections are known. Amabili and Kwak (Amabili, 1996; Amabili and Kwak, 1996; Amabili et al., 1996; Kwak, 1997) did abundant valuable work to investigate the mixed boundary problems, in which circular and annular plates, free surfaces and rigid wall interfaces and the influences of water depth are considered. With the velocity potential obtained based on Green method, Kwak (1996) investigated the natural frequencies of rectangular plates in contact with infinite water. Then, Amabili (Amabili, 2000, 2001; Amabili and Kwak, 1999) studied the influences of the surface waves on the vibrations of a vertical, simply supported circular cylindrical shell partially filled by an incompressible liquid and circular plates resting on infinite liquid domain or completely covering the liquid in a circular cylindrical tank, and the sloshing modes were investigated deeply and systematically. Recently, Kwak et al. (2011) studied the free flexural vibration of a hung clamped-free cylindrical shell partially submerged in a fluid, and an analytical approach based on Rayleigh quotient, Ritz expansion and Galerkin method is developed by Askari et al. (2011) to investigate the effects of a rigid internal body on bulging and sloshing frequencies and modes of a cylindrical container partially filled with a fluid. Zhou et al. (Cheung and Zhou, 2000, 2002; Zhou and Cheung, 2001; Zhou and Ji, 2005) studied the vibratory characteristics of the rectangular plates and circular plates that respectively placed at the bottom of the cuboid and columniform containers filled with water, in which the motion of water is expressed using the velocity potential, and the influences of free surface and rigid top were investigated. Ergin and Uğurlu (2003) investigated natural frequencies and mode shapes of cantilever plates partially in contact with a fluid, in which the pressure of water are calculated by use of boundary-integral equation method and the method of images. It is found that the results are in good agreement with the experiment results through the added mass coefficients were obtained using the dry modes information. Recently, Jeong (2006) studied a linear hydroelastic vibration of two annular plates coupled with a bounded fluid based on the Rayleigh–Ritz method and the finite Hankel transform.

In above mentioned research, many researchers (Amabili, 1996; Amabili and Kwak, 1996, 1999; Amabili et al., 1996; Kwak, 1997; Montero De Espinosa and Gallego-Juarez, 1984) studied the influence of water on the natural frequencies based on the hypothesis that the mode shapes of the plate vibrating in contact with a fluid (wet mode shapes) are identical with those of the plate vibrating in a vacuum (dry mode shapes). This technique simplifies the analysis, however, the influences of water on the mode shapes are ignored and the precision of natural frequencies may be reduced. To investigate the vibrations of the FSI systems to obtain the wet mode shapes and the corresponding natural frequencies more accurately, the Rayleigh–Ritz method, in which the added kinetic energy of water is included in the procedure, was introduced by Zhu (1995) and widely used in the following investigations (Amabili, 1996, 2000, 2001; Amabili and Kwak, 1996; Amabili et al., 1996; Cheung and Zhou, 2000; Kwak, 1996; Zhou and Cheung, 2000, 2001). The comparison shows that the fundamental frequency is well estimated by the method based on the hypothesis that the wet mode shapes are identical with the dry mode shapes. However, for higher modes, the results obtained based on that hypothesis are less accurate, and the Rayleigh–Ritz method is necessary (Amabili, 1996; Amabili and Kwak, 1996; Amabili et al., 1996; Kwak, 1996).

To analyze the vibration of cracked plates, the common methods include finite element method (FEM) based on fracture mechanics (Qian et al., 1991) and the Ritz method (Huang and Leissa, 2009; Kim and Jung, 2004; Leissa et al., 1993). Combining the corner functions introduced by Williams (1952), Leissa et al. (1993) analyzed the influences of the V-notches on the vibrations of the completely free circular plates based on the Ritz method. Recently, Huang and Leissa (2009) proposed a new set of corner functions, and faster convergence speed were obtained during the dynamic analysis of rectangular plates with side cracks and simply supported.

In some cases, the cracks may appear on the structures in contact with liquid due to the alternating pressure load, such as the cracks on the blades and vane of hydraulic turbine (Gummer and Hensman, 1992), which may enhance the vibration and lead to catastrophic failure. To analyze the free vibration of this kind of structures, the influences of both crack and liquid should be included in the modeling. However, the corresponding research is insufficient. Shi et al. (2004) calculated the natural frequencies corresponding to each mode of a circular clamped baffled plate with a crack and in contact with water. However, the results were obtained through iteration and the influences of water on the mode shapes were ignored. Based on the Rayleigh–Ritz method, Si et al. (2011, 2012) investigated the free vibrations of circular plates with a side crack and in contact with water on one side, in which the cracks start at the boundary and develop along the radial direction.

In view of the fact that no analysis has been found in the literature for the dynamic characteristics of a cracked rectangular plate interacting with water, the free vibrations of baffled rectangular plates with side cracks and in contact with water on one side for various boundary conditions are investigated based on the Rayleigh–Ritz method in the present paper. After the validity and accuracy of the proposed method are demonstrated in the convergence studies, some numerical results are presented.

2. Methodology

The plate considered here is a rectangular plate with a single side crack and in contact with water on one side, as shown in Fig. 1(a). The plate is located at a hole of an infinite rigid plane wall, and the dimension of water is infinite. The origin of the coordinate system is located at the crack tip and the crack develops along the line $\theta=0^\circ$. The dimensionless distance