INTERNATIONAL JOURNAL OF REFRIGERATION 34 (2011) 567–572

available at www.sciencedirect.com

ScienceDirect

journal homepage: www.elsevier.com/locate/ijrefrig

Performance optimization of an irreversible Carnot refrigerator with finite mass flow rate

Jianlin Yu, Yuanyuan Zhou, Ying Liu*

Department of Refrigeration and Cryogenic Engineering, School of Energy and Power Engineering, Xi'an Jiaotong University, Xi'an 710049, China

ARTICLE INFO

Article history:
Received 15 June 2010
Received in revised form
17 August 2010
Accepted 27 October 2010
Available online 30 October 2010

Keywords:
Carnot
Refrigerator
Irreversible
Thermodynamics
Optimization

ABSTRACT

The present study develops a theoretical model for the optimization of an irreversible Carnot refrigerator subject to a constraint of finite mass flow rate, which includes internal as well as external irreversibilities. By introducing two dimensionless parameters as indicative of the mass flow rates for the refrigerator, the new model allows detailed analyses on the finite mass flow rate allocation problem of working fluids among the hot- and cold-side heat exchangers of Carnot refrigerators. The analytical solutions of the maximum coefficient of performance (COP) for irreversible Carnot refrigerators are obtained under the equivalent of the finite-flow rate constraint. Furthermore, the influences of major parameters on the maximum COP and the corresponding mass flow rate allocation are examined and shown by numerical examples. The obtained results may provide a theoretical guidance for the optimal design and operation of real refrigerators.

Optimisation de la performance d'un réfrigérateur Carnot irréversible avec un débit massique fini

Mots clés : Carnot ; Réfrigérateur ; Irréversible ; Thermodynamique ; Optimisation

1. Introduction

Finite time thermodynamics (FTT) has become the premier method of thermodynamic analysis in thermodynamic cycles and devices since an endoreversible model of Carnot cycle was considered and developed during the 1950s and 1970s (Curzon and Ahlborn, 1975; Chen et al., 2001a; Feidt et al., 2007). The methods of FTT are used in the optimization of real devices and processes, subject to finite-size and finite-time constraints (Bejan, 1996). Over the past years, the analysis and optimization of various heat engines and refrigerators for different optimization objectives have made tremendous progress by employing FTT concepts (Agnew and Ameli, 2004; Aragón-González et al., 2003; Chen, 1998, 2001; Chen et al., 2001b; Gordon et al., 1997; Petre et al., 2009; Tyagi et al., 2002; Ust, 2009; Wang et al., 2005; Zhang et al., 2009). These developments have provided guidelines for the optimal design and operation of various real refrigerators.

* *Corresponding author*. Tel.: +86 29 82668738; fax: +86 2982668725.
E-mail address: yujl@mail.xjtu.edu.cn (J. Yu).
0140-7007/$ – see front matter © 2010 Elsevier Ltd and IIR. All rights reserved.
doi:10.1016/j.ijrefrig.2010.10.010

Nomenclature

c_p	specific heat (kJ kg^{-1} K^{-1})
COP	coefficient of performance
HEX	heat exchanger
I	irreversibility factor
$\dot{m}$	mass flow rate (kg s^{-1})
$\dot{Q}$	heat transfer rate (W)
q	heat transfer per unit mass (kJ kg^{-1})
R	dimensionless parameter
Δs	entropy change (kJ kg^{-1} K^{-1})
T	refrigerant temperature (K)
T'	inlet temperature of external working fluid (K)
T''	outlet temperature of external working fluid (K)
UA	thermal conductance
x	relative mass flow rate allocation ratio
z	real mass flow rate ratio

Greek symbols

ε	effectiveness factor of heat exchanger

Subscripts

c	cold-side
h	hot-side
max	maximum
opt	optimal
r	refrigerant

The Carnot refrigeration cycle is one of the important cycle models of refrigerators. Also by using the FTT, a series of investigations related to the optimal performances of irreversible Carnot refrigeration cycles have been carried out (Agnew et al., 1997; Chiou et al., 1995; Feidt et al., 2007; Ust and Sahin, 2007; Velasco et al., 1997; Wu et al., 1996). These optimizations for cooling capacity and coefficient of performance (COP) are carried out subject to various physical constraints that are in fact responsible for the irreversible operation of the Carnot refrigerator. Most of the studies of physical constraints required to bind the optimum solution have mainly focused on the finite-rate heat transfers between the cycle working fluid (refrigerant) and the external working fluids (heat reservoirs), together with thermal conductance constraint. However, there exist the constraints of finite mass flow rate in real processes besides the constraint of thermal conductance. For a real irreversible Carnot refrigerator with the finite-size considerations, the mass flow rates of the refrigerant and external working fluids must be subject to certain constrains and cannot be specified arbitrarily. Thus, it is important to analyze the effects of finite mass flow rates on the optimal performance of an irreversible Carnot refrigerator.

In the present study, we introduce another approach to deal with the optimization of irreversible Carnot refrigerators. A cycle model for Carnot refrigerators that accounts for the irreversibilities due to the heat transfer and internal dissipation of the refrigerant is established. The performances of the resulting model are optimized in finite mass flow rate and finite size. In the optimization of the model, two new dimensionless parameters are defined, which are related to the fluid mass flow rates of the hot- and cold-side fluids. Furthermore, the summation of the dimensionless mass flow rate of the hot- and cold-side fluids is constrained and the allocation of this constrained mass flow rate is optimized. The objective of this study is to investigate the maximum COP of irreversible Carnot refrigerators to obtain analytical expressions for the relevant optimum performance parameters.

2. Irreversible cycle model for Carnot refrigerators

It is known that in real refrigerators there exist three types of irreversibility: finite-rate heat transfers, heat leak and internal dissipation of the refrigerant. In the following analysis, for simplicity the heat leak between the heat reservoirs is assumed to be negligible. The refrigerator model with two finite-size heat exchangers and finite mass flow rate fluids is shown in Fig. 1 (a). In such a model, the irreversible Carnot refrigeration cycle inside the refrigerant consists of two non-isentropic processes (compression and expansion) and two isothermal processes (heat rejection and absorption) as shown in the T-s diagram of Fig. 1 (b). We only consider refrigerator operation at steady state. The method of the effectiveness-number of transfer units (NTU) for the two heat exchangers is used to establish the irreversible refrigerator model. Thus, the heat transfer rates can be written as

$$\dot{Q}_c = T_c \Delta s_c \dot{m}_r = \varepsilon_c \dot{m}_c c_{pc} (T'_c - T_c) \tag{1}$$

$$\dot{Q}_h = T_h \Delta s_h \dot{m}_r = \varepsilon_h \dot{m}_h c_{ph} (T_h - T'_h) \tag{2}$$

where ε_c and ε_h are the effectiveness factors of the cold- and hot-side heat exchangers, respectively, and can be expressed as (Chiou et al., 1995)

$$\varepsilon_c = \frac{T'_c - T''_c}{T'_c - T_c} = 1 - \exp\left(-\frac{(UA)_c}{\dot{m}_c c_{pc}}\right) \tag{3}$$

$$\varepsilon_h = \frac{T''_h - T'_h}{T_h - T'_h} = 1 - \exp\left(-\frac{(UA)_h}{\dot{m}_h c_{ph}}\right) \tag{4}$$

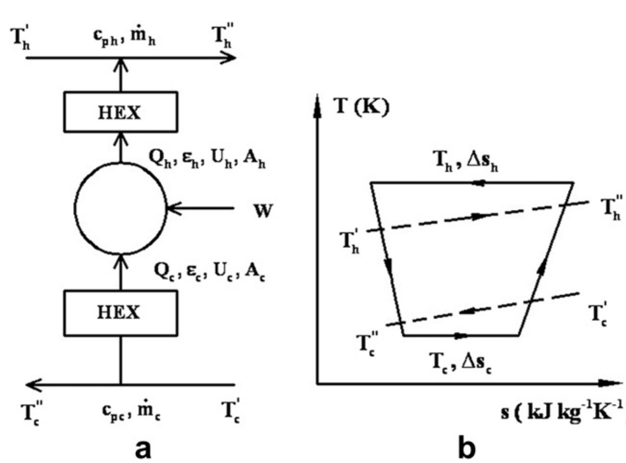

Fig. 1 – (a) A schematic diagram of an irreversible refrigerator; (b) The T-S diagram of an irreversible cycle.