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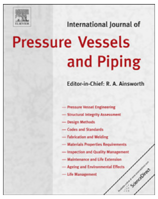

Limit state equations for circular cross sections subjected to combined loading

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ABSTRACT

Quick information about the capacity of any cross section for combined loading is very useful information at the early stages of engineering design. These combined loads can be pressure, axial force, bending moment and torsion, depending on application of cross section. Interaction formula based on limit loads is a very useful equation to estimate the capacity of any cross section for a combination of loads. For hollow circular cross section the complexity of interaction formulae depends on definition of limit state and type of loads. Interaction formulae for circular cross section with different limit state assumptions are easily available in literature for pressure, axial force and bending moment. This paper proposes interaction formulae for circular cross section with additional term of torsional moment at elastic limit load.

1. Introduction

1.1. Introduction

Hollow circular section is very commonly used shape in oil & gas industry for pressurized or non-pressurized parts such as pipe lines, jumpers, risers or structural members. In all applications it either carries or transfers the installation, environmental or operational loads. These loads are typically internal/external pressure, bending moment, axial force and torsion. Calculation of capacity of hollow cross section for individual load is relatively straight forward. Detailed analytical solutions are available in literature for all type of loads based on theory of elasticity.

However in all practical engineering applications circular cross sections are subjected to combination of loads. Predicting load capacity analytically, for combination of loads, is challenging. Some of the difficulties are in defining, 'what is failure'. Is it 'yielding at a point or through thickness' or 'collapse of complete cross section'. Initial yielding location depends on the type of load and hence relative magnitude of loads influences failure mechanism. Also there are other difficulties related to geometry or additional loads like temperature variation which calls for analysis during detail engineering.

However, during tendering or initial stage of project, early information about maximum load carrying capacity of cross section for different combination of loads is very useful information. It helps design engineers to freeze the concept, or to give interface loads as 'advance information' to other departments. Interface load carrying capacity with moderate accuracy is required to start parallel engineering activities. Load interaction formula is useful equation to extract this information.

Load interaction formulae for pipe sections subjected to pressure, bending moment and axial loads are already available in literature. In many cases like pipelines, torsional load is negligible or very low, and hence neglected. However there are specific applications like subsea jumpers which are subjected to all the four loads and where the effect of torsion on limit state cannot be ignored. To predict failure in such cases we need load interaction equations which are not available in any literature. This paper proposes load interaction formulae to predict failure of pipes with additional term of torsional load in combined loading along with pressure, bending moment and tension. The proposed equations are developed using the simple bending theory, torsion theory and the thick and thin cylinder theory and hence applicable to all materials satisfying the assumptions of these theories.

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Nomenclature

P, P_i	Internal Pressure
P_o	External pressure
N	Axial thrust
M	Bending moment
T	Torsion
P_c	Pressure capacity
N_c	Axial capacity
M_c	Bending moment capacity
T_c	Torsion capacity
r_i	Inner radius
r_o	Outer radius
r	Radius
r_m	Mean radius
d_i	Inner diameter
d_o	Outer diameter
d	Diameter
t	Thickness
σ_y	Yield strength
σ_{xx}	Normal stress in x-direction
σ_{yy}	Normal stress in y-direction
σ_{zz}	Normal stress in z-direction
τ_{xy}	Shear stress in y-direction
τ_{yz}	Shear stress in z-direction
τ_{zx}	Shear stress in x-direction
τ	Shear stress
$\tau_{\max}$	Maximum shear stress
σ_n	Axial stress because of axial thrust
σ_b	Bending stress
σ_{lp}	Normal stress because of the end cap force
σ_h	Hoop stress
σ_l	Longitudinal stress
σ_r	Radial stress
σ_1	First principal stress
σ_2	Second principal stress
σ_3	Third principal stress
I	Moment of inertia
J	Polar moment of inertia
y	Distance of extreme fibre from the neutral axis
SCL	Stress classification line
S	Stress tensor

1.2. Limit state

The limit state approach predicts the failure at the following limits-elastic limit, elasto − plastic limit, cross-section limit and collapse limit. These limits are represented by the points a, b, c and d respectively as shown in Fig. 1. The dotted lines represent the failure caused by premature buckling.

Elastic limit load (point a) is the load required for the first yield to occur.

Elasto-plastic limit load (point b) is the load required to cause the through thickness failure of the pipes.

Cross-section limit load (point c) is the load required to cause the complete failure of the cross section or the load needed for the formation of the first plastic hinge.

Collapse limit load (point d) is the load required for the formation of sufficient plastic hinges which cause the collapse of the structure. This limit includes the effect of strain hardening and geometrical strengthening.

Premature failure may also occur because of local bucking when the external hydrostatic pressure exceeds the internal pressure. Bending moment is a critical parameter in buckling failure. The applied bending energy will accumulate at the local buckle and this will eventually lead to geometrical collapse of the pipe once the maximum bending resistance of the pipe is reached.

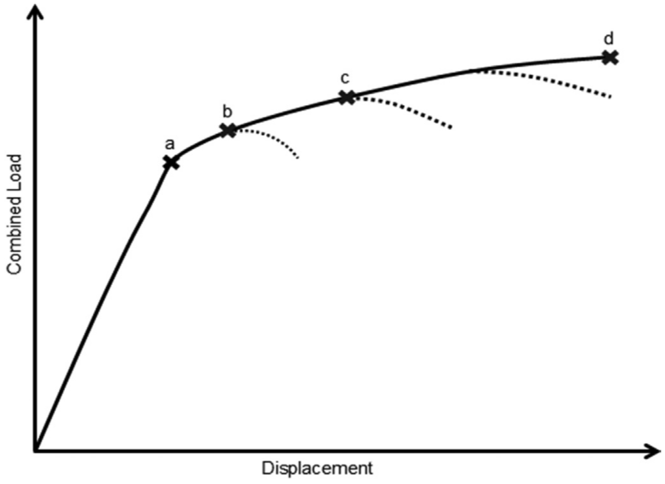

Fig. 1. Combined Load vs Displacement relationship.

It is easy to predict the pipe failure when it is subjected to pure bending or pure torsion or pure axial thrust or pure internal pressure. But for the case of combined loading, the interaction of various loads is to be understood. Amran [1] has proposed such load interaction relation taking a combination of two loads at a time (axial-pressure, torsion-pressure and Bending moment-pressure). Bai [2] has proposed equations to calculate the ultimate moment capacity of a pipe subjected to combined loading of axial thrust + bending + pressure. Finn [3] has also proposed load interaction relations taking combined loading of axial + bending + pressure. In all the references mentioned above, the effect of torsional load in combined loading considering all four loads-pressure, bending moment, axial load and torsion has not been accounted for. In the literature which we have studied, there are no load interactions equations available which take into account all the four loads acting on the pipe and predict its failure. This paper presents load interaction relations for thick ($D/t < 20$) and thin ($D/t > 20$) pipes for a combined loading taking all four loads-axial thrust + bending moment + internal pressure + torsion into account and thus predicting the failure of pipes.

The formulae proposed in this paper are for the elastic limit load. The pipe is considered to fail after the first yield has occurred. This is a conservative assumption, and as seen in Fig. 1 the pipe has much more capacity left after point (a).

2. Derived formulae

2.1. Assumptions

- The effect of transverse shear is neglected as its contribution is insignificant in combined loading.
- The radial stress in thick and thin cylinders is assumed to be zero for mathematical simplifications. The effect of this assumption in predicting elastic limit load is seen using FEA and presented in chapter 3.