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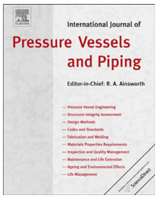

# Effect of local wall thinning on shakedown regimes of pressurized elbows subjected to cyclic in-plane and out-of-plane bending

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ABSTRACT

The current research utilizes a direct non-cyclic technique to generate elastic shakedown domains for thinned-wall 90° elbows. The elbows are subjected to simultaneous steady internal pressures and cyclic in-plane and out-of-plane bending moments. Wall thinning is located at the intrados, extrados, and crown once at a time. Effects of thinning depth and thinning location under both cyclic in-plane and out-of-plane bending modes are investigated. Generated shakedown boundaries are compared to those corresponding to sound elbows. Elbows subjected to out-of-plane bending moments revealed relatively higher shakedown domains compared to corresponding elbows subjected to in-plane bending. It is generally noticed that thinning at the intrados or crown has more severe effect on reducing elbows shakedown domains as compared to thinning at the extrados for both in-plane and out-of-plane bending modes.

## 1. Introduction

Sudden changes in fluid flow direction result in significant local wall thinning due to erosion and erosion/corrosion in elbows as compared to straight pipes as reported within JSME [1] and HSE [2]. Corrosion is generally referred to as the deterioration of metals due to chemical interaction with the surrounding environment. Erosion is a form of wear which results in gradual removal of surface metal. Erosion occurs due to bombardment of solid particles, flowing along the pressurized fluid stream due to centrifugal forces, with elbow walls. Erosion/corrosion is an accelerated corrosion mechanism due to corrosive fluid flow against metal surface. Also known as flow accelerated corrosion (FAC), erosion/corrosion occurs due to the development of excessive fluid turbulence. Upon exceeding the elastic shakedown (SD) domain, elbows are vulnerable to low cycle fatigue and/or ratcheting modes of failure. Low cycle fatigue failure occurs due to reversed plasticity (RP) whereas ratcheting (R) occurs due to incremental accumulation of plastic strain associated with every load cycle. The current research focuses on generation of elastic SD domains for pressurized elbows with local wall thinning with various depths and locations subjected to cyclic in-plane closing (IPC), in-plane opening (IPO), and out-of-plane (OP) bending moments. Operation within the elastic SD domain ensures elastic cyclic steady state, following limited accumulation of plastic strain during the initial load cycles, at critical points within the elbow structure. Elbows subjected to cyclic loading conditions are typically designed to operate within the SD domain, and it is crucial to generate data quantifying the effects of wall thinning on SD domain areas.

## 2. Review of literature

Successful attempts to formulate a lower bound shakedown (SD) theorem began in the late twenties [3] and early thirties [4] of the 20th century. However, the first comprehensive form of the lower bound SD theorem was presented in 1936 by Melan [5] which states the following: “*For a given load set P, if any distribution of self-equilibrating residual stresses can be found, assuming perfect plasticity, which when taken together with elastically calculated stresses, constitute a system of stresses within the yield limit, then P is a lower bound shakedown load set and the structure will*

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*Abbreviations:* ASME, American Society of Mechanical Engineers; EPP, Elastic-Perfectly-Plastic; FE, Finite Element; HSE, Health and Safety Executive; IPC, In-Plane Closing; IPO, In-Plane Opening; JSME, Japan Society of Mechanical Engineers; LVDT, Linear Variable Displacement Transducer; OP, Out-of-Plane; R, Ratcheting; RP, Reversed Plasticity; SD, Shakedown.

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**Nomenclature**

| | |
|---|---|
| $E$ | modulus of elasticity (Young's Modulus) |
| $F$ | reaction force |
| $L$ | equivalent axial thinning length defined at crown |
| $M$ | bending moment |
| $P$ | internal pressure |
| $U$ | nodal displacement |
| $UR$ | nodal rotation |
| $D_o$ | elbow outer diameter |
| $2L_{Circ}$ | circumferential thinning width |
| $L_{Axial}$ | axial (meridional) thinning length |
| $L_{Pipe}$ | length of attached straight pipes |
| $M_L$ | elbow in-plane bending collapse moment |
| $P_L$ | limit pressure of sound elbow |
| $P_L^{th}$ | limit pressure of thinned-wall elbow |
| $R_b$ | bend radius of elbow |
| $M_i$ | bending moment magnitude at a finite element solution increment ($i$) |
| $M_{ref}$ | arbitrary reference moment load magnitude |
| $S_u$ | material ultimate strength |
| $S_Y$ | material yield strength |
| $i$ | finite element elastic–plastic solution increment |
| $k$ | material strength coefficient |
| $n$ | material strain hardening exponent |
| $r$ | elbow mean radius |
| $t$ | elbow nominal wall thickness |
| $x$ | arbitrary positive integer |
| $t_p$ | elbow minimum (remaining) wall thickness |
| $\delta$ | displacement |
| $\lambda$ | bend characteristic |
| $2\theta$ | circumferential thinning angle |
| $\nu$ | Poisson's ratio |
| $\sigma$ | true stress |
| $\varepsilon_p$ | true plastic strain |
| $\sigma_E$ | elastic stress components |
| $\sigma_{ELPL}$ | elastic–plastic stress components |
| $\sigma_{unload}$ | stress components determined during moment unloading at the end of every elastic–plastic solution increment ($i$) |
| $\sigma_{unload_{eq}}$ | equivalent stress calculated utilizing determined stress components ($\sigma_{unload}$) during moment unloading at the end of every solution increment ($i$) |

*shakedown*". The ASME Boiler and Pressure Vessel Code [6] defines elastic SD as: "*the absence of significant progressive cyclic inelastic deformation*".

Chen X. et al. [7] performed ratcheting tests and finite elements (FE) simulations adopting advanced material constitutive models on pressurized sound carbon steel elbows by means of cyclic application of reversed in-plane bending. The FE results showed reasonable correlation with the recoded experimental outcomes. Maximum ratchet strain occurred at the crowns in the circumferential direction. Circumferential ratcheting strain was observed at intrados in some individual elbows. No ratcheting strain was observed at the extrados for all tested elbows. Utilizing a direct non-cyclic technique, Abdalla et al. [8] verified the SD boundary generated by Chen X. et al. [7] and good correlation was obtained within the medium to high steady pressure range. Ratcheting tests were conducted by Vishnuvardhan S. et al. [9] on grade 304LN stainless steel sound pressurized elbows subjected to cyclic in-plane bending moments. It is observed that ratcheting in the circumferential direction at the crown and intrados is more significant than in the axial or meridional direction at both locations [9]. Additionally, ratchet strain accumulation at the crown was found to be predominant as compared to the intrados [9]. Similar to Chen X. et al. [8] observations, no significant ratcheting strain is observed at the extrados. Yahiaoui et al. [10] tested sound pressurized carbon and stainless steel elbows under cyclic in-plane bending moments. Crown ratcheting in the circumferential direction is reported to be greater than in the axial direction and rapidly increased once initiated.

Varelis et al. [11] conducted experimental tests and FE analyses on low pressurized sound steel elbows to investigate their responses under simulated seismic cyclic bending effects. Number of cycles to failure by low cycle fatigue was determined and results of conducted FE analyses illustrated good agreement with recorded test results. Upon comparison with ASME B31.3 and EN 13480-3 design standards for occasional loading conditions, Varelis et al. [11] results highlighted lack of design provisions within the low cycle fatigue range. Chen H. et al. [12] generated SD domains utilizing the Linear Matching Method (LMM) for sound pressurized elbows subjected to cyclic IPO, IPC, and reversed bending modes. Cyclic through wall temperature difference of pressurized elbows was also studied to investigate the effect of thermal stresses on generated SD domains. It is reported that application of cyclic thermal load resulted in significant reduction in SD domains [12]. Wood et al. [13] conducted experimental tests and FE analyses on 90° sound pressurized single unreinforced mitred bend subjected to cyclic IPC bending. The bend failed due to a through-thickness crack leading to leakage resulting from combined ratcheting and low cyclic fatigue responses.

Kim J. et al. [14] conducted experimental tests to investigate to effect of monotonic IPC and IPO bending on the burst pressure capacity of elbows with thinning at the intrados/extrados/full-circumference. It is observed that bursting occurred due to axial cracking following bulging irrespective of bending direction or thinning location [14]. Lee et al. [15] conducted real scale tests on 24 elbow specimens with locally thinned walls to investigate the failure behavior of pressurized carbon steel elbows subjected to both IPC and IPO monotonic bending till collapse. Locally thinned areas were introduced via machining at the extrados/intrados. Test results for IPO revealed increasing load-deflection trend of the plotted curves till test termination thereby illustrating hardening effect. Contrarily, IPC curves illustrated initially increasing load-deflection trend till failure by ovalization followed by decreasing load with increasing displacement thereby exhibiting softening effect. Lee et al. [15] test samples are modeled within the current research to validate the results of developed nonlinear FE model employed to generate shakedown boundaries for elbows with locally thinned walls.

Takahashi et al. [16,17] conducted experimental tests and FE analyses to investigate low cycle fatigue of thinned-wall carbon steel elbows subjected to cyclic in-plane bending in the absence of internal pressure. Thinning is located at the extrados/crown/intrados. Locations of crack initiation and propagation direction were adequately predicted via 3D elastic–plastic FE analyses. It is concluded that elbows with thinning at the extrados have stronger resistance to low cycle fatigue failure compared to elbows with thinning at the crown and intrados. Low cycle fatigue lives predicted via FE analyses based on strain criteria correlated well with test results [17].