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The maximum possible stress intensity factor for a crack in an unknown residual stress field

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ABSTRACT

Residual and thermal stress fields in engineering components can act on cracks and structural flaws, promoting or inhibiting fracture. However, these stresses are limited in magnitude by the ability of materials to sustain them elastically. As a consequence, the stress intensity factor which can be applied to a given defect by a self-equilibrating stress field is also limited. We propose a simple weight function method for determining the maximum stress intensity factor which can occur for a given crack or defect in a one-dimensional self-equilibrating stress field, i.e. an upper bound for the residual stress contribution to K_I. This can be used for analysing structures containing defects and subject to residual stress without any information about the actual stress field which exists in the structure being analysed. A number of examples are given, including long radial cracks and fully-circumferential cracks in thick-walled hollow cylinders containing self-equilibrating stresses.

1. Introduction

Residual stresses strongly influence elastic fracture, but can be difficult and time-consuming to measure [1]. Furthermore, accurate prediction of the residual stresses which result from manufacturing operations is challenging and usually involves the use of elastic–-plastic finite element modelling. As a consequence, fracture-mechanics-based integrity assessment of components and structures containing residual stresses is often hampered by a lack of reliable residual stress data for the object being analysed [2]. When the residual stress distribution is not known, it is necessary to use conservative assumptions regarding the nature of the residual stress field. This results in cautious and often highly conservative assessments of structural integrity for residual stress-bearing structures, often leading to safe plant being taken out of service earlier than required and at significant cost.

The most important way in which residual stress information is used in the assessment of structural integrity is to calculate the contribution of the residual stress state to the Mode I stress intensity factor K_I for a given defect in the component or structure being assessed. In the absence of residual stress data, most structural integrity assessment procedures such as BS7910 [3] and R6 [4] specify that a conservative estimate of the residual stress state should be used. This can be taken from a handbook of distributions which provide an upper bound to previous experimental data for common geometries. In cases for which such bounding distributions are not available, it is often assumed that the crack-normal stress is uniformly tensile and at yield magnitude [5]. The use of conservative estimates of the real residual stress distribution in a component simplifies analysis but can lead to unrealistically large estimates of K_I, especially for deeper cracks and defects. The fundamental issue is that conservative estimates of the residual stress distribution must be biased towards a tensile state of crack-normal stress. However, for deeper cracks the effect of this tensile bias on the resulting calculated stress intensity factor can become very large, resulting in estimates of the residual stress contribution to K_I which are unrealistically high.

A useful simplification is to decompose the distribution of crack-normal stress $\sigma_{yy}(x)$ on the section containing the crack or defect into three components [6,7]: a membrane component σ^m, a bending component σ^b and a self-equilibrating component σ^{se}, as shown in Fig. 1. In this example the membrane component σ^m is finite; a net force across the section is balanced by forces elsewhere in the structure. However, residual stresses often occur in components where no net force over the complete section is possible either because the component being considered is free-standing, or because it is very stiff relative to the rest of the structure, or because it is known to fit into the structure without any interference.

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Nomenclature

a	crack length
a_t	crack length below which a uniformly tensile stress of magnitude σ^{lim} in the direction normal to the crack can exist over the entire length of the prospective crack in a self-equilibrating stress distribution
A	total sectional area
b	overall section width or characteristic length
c	distance of a point force from the crack mouth (or from the centre of a symmetric crack)
K_I	mode I stress intensity factor
K_I^{max}	maximum possible Mode I stress intensity factor which can be generated by a residual stress
m	weight function for K_I for unit normal crack face point loading
m^{lim}	limiting value of the weight function used for assigning tensile and compressive stress regions
(n_k)	sequence of indices of the largest elements of $\frac{m(x_n)}{r(x_n)}$
P	normal point force applied to crack face(s)
q	welding torch power
Q	weld heat input per unit length per unit section thickness
r	radial distance (cylindrical coordinates)
r_a	radius from the axis of the pipe to the tip of a circumferential crack
r_i	pipe internal radius
r_o	pipe external radius
v	velocity of welding torch
x	distance in crack extension direction (Cartesian coordinates)
x_n	set of N uniformly-spaced points in the interval $0 \leq x \leq a$
y	distance in the direction normal to the crack plane (Cartesian coordinates)
z	axial distance (cylindrical coordinates)
θ	azimuth (cylindrical coordinates)
σ^b	bending component of a sectional residual stress distribution
σ^m	membrane component of a sectional residual stress distribution
σ^{lim}	maximum possible stress in the direction normal to the crack plane
σ^{se}	self-equilibrating component of a sectional residual stress distribution
$\mathbb{X}_c$	domain of compressive crack-normal stress on the prospective crack plane
$\mathbb{X}_t$	domain of tensile crack-normal stress on the prospective crack plane

Previously, several authors have examined the relationship between a periodic distribution of self-equilibrating crack-normal stress and the maximum values of stress intensity factor for a crack introduced into in this stress field [8–10]. This demonstrated the diminishing contribution of lower-wavelength periodic stress terms to K_I. However, for the purpose of structural integrity assessment it is useful to also consider the maximum possible value of stress intensity factor that could be applied by a self-equilibrating stress acting on a particular crack.

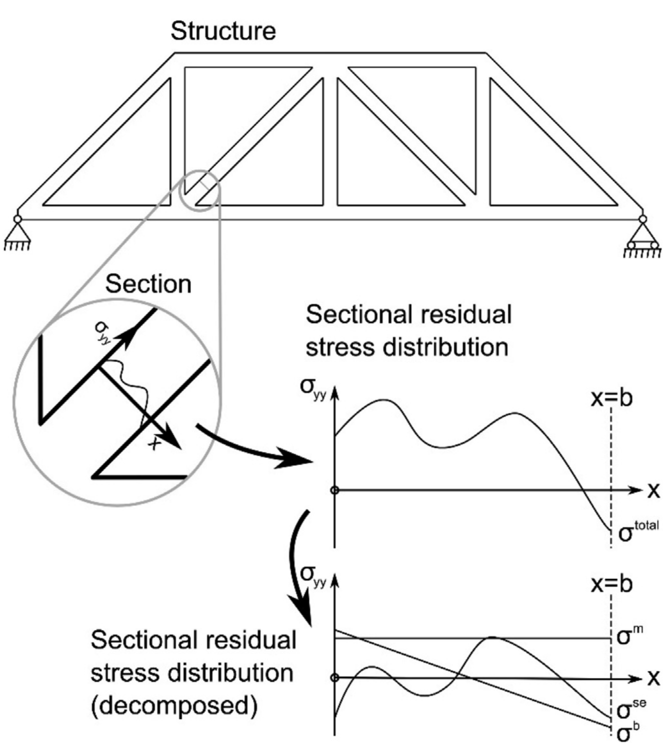

Fig. 1. Residual stress across a section of a component in a larger structure, and decomposition of the stress distribution into membrane, bending and self-equilibrating components.

The aim of this work is to determine upper-bound values for the residual stress contribution to K_I which can be used when the residual stress distribution in the defect-containing component is not known. We consider objects containing one-dimensional residual stress distributions which produce no net section force, and evaluate the maximum stress intensity factor K_I^{max} which is possible for a given crack due to the existence of a stress field that is self-equilibrating across the section on which the crack lies. Previously, we have proposed a method based on inverse eigenstrain analysis for approximating the maximum possible contribution to K_I of a unknown residual stress field [11]. This method can be applied to components containing complex residual stress distributions, but requires the use of Finite Element Analysis (FEA) and only yields an approximation of the K_I upper bound. Here, we describe a simplified method of determining the maximum possible residual stress contribution to K_I which is applicable to one-dimensional stress distributions with $\sigma^m = 0$. This method is based on weight function analysis, so for many geometries it can be performed using existing weight function solutions without the need for additional FEA results.

2. Method of analysis

2.1. Force and moment equilibrium

Fig. 1 shows the (one-dimensional) distribution of residual stress inside a component which forms part of a larger structure, and how it can be decomposed into membrane (σ^m), bending (σ^b) and self-equilibrating (σ^{se}) parts. For a finite membrane stress to occur, the rest of the structure must impart a force on the component, i.e. the component must only fit into the structure with interference. Likewise, for the stress distribution shown in Fig. 1 a finite bending component can only occur if the rest of the structure