International Journal of Pressure Vessels and Piping 134 (2015) 92–100

Contents lists available at ScienceDirect

International Journal of Pressure Vessels and Piping

journal homepage: www.elsevier.com/locate/ijpvp

The ratchet–shakedown diagram for a thin pressurised pipe subject to additional axial load and cyclic secondary global bending

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ARTICLE INFO

Article history:
Received 17 March 2015
Received in revised form
20 August 2015
Accepted 28 August 2015
Available online 1 September 2015

Keywords:
Shakedown
Ratcheting
Pipe

ABSTRACT

The ratchet and shakedown boundaries are derived analytically for a thin cylinder composed of elastic-perfectly plastic Tresca material subject to constant internal pressure with capped ends, plus an additional constant axial load, F, and a cycling secondary global bending load. The analytic solution is in good agreement with solutions found using the linear matching method. When F is tensile, ratcheting can occur for sufficiently large cyclic bending loads in which the pipe gets longer and thinner but its diameter remains the same. When F is compressive, ratcheting can occur in which the pipe diameter increases and the pipe gets shorter, but its wall thickness remains the same. When subject to internal pressure and cyclic bending alone ($F = 0$), no ratcheting is possible, even for arbitrarily large bending loads, despite the presence of the axial pressure load. The reason is that the case with a primary axial membrane stress exactly equal to half the primary hoop membrane stress is equipoised between tensile and compressive axial ratcheting, and hence does not ratchet at all. This remarkable result appears to have escaped previous attention.

1. Introduction

A structure subject to two or more types of loading, at least one of which is primary and at least one of which is cycling, may potentially accumulate deformation which increases cycle on cycle. This is ratcheting. Rather less severe loading may result in parts of the structure undergoing plastic cycling, involving a hysteresis loop in stress–strain space, but without accumulating ratchet strains. Still less severe loading may result in purely elastic cycling, perhaps after some initial plasticity on the first few cycles. This is shakedown. It is desirable that engineering structures be in the shakedown regime since ratcheting is a severe condition leading potentially to failure. The intermediate case of stable plastic cycling may be structurally acceptable but will involve the engineer in non-trivial assessments to demonstrate acceptability, probably involving the possibility of cracks being initiated by the repeated plastic straining (by fatigue).

Deciding which of the three types of behaviour results from a given loading sequence on a given structure is, therefore, of considerable importance. Unfortunately ratcheting/shakedown problems are difficult to solve analytically in the general case. However, analytical solutions for sufficiently simple geometries and loadings do exist. One of the earliest, and undoubtedly the most influential, of these is the Bree problem, Ref. [1]. Bree's analytic solution addresses uniaxial loading of a rectangular cross section in an elastic-perfectly plastic material, the loading consisting of a constant primary membrane stress and a secondary bending load which cycles between zero and some maximum. When normalised by the yield stress, the primary membrane stress is denoted X whilst the normalised secondary elastic outer fibre bending stress range is denoted Y.

The ratchet boundary is defined as the curve on an X, Y plot above which ratcheting occurs. Similarly, the shakedown boundary is defined as the curve on the X, Y plot below which shakedown to elastic cycling occurs. The two curves may or may not be separated by a region of stable plastic cycling. In obvious notation, the three types of region are denoted R, S and P. Variants on the Bree problem which have been solved analytically include, (i) the case when the primary membrane load also cycles, either strictly in-phase or strictly in anti-phase with the secondary bending load, Refs. [2–4], (ii) the Bree problem with different yield stresses at the two ends of the load cycle, Ref. [5], and, (iii) the Bree problem with biaxial

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http://dx.doi.org/10.1016/j.ijpvp.2015.08.008

stressing of a flat plate, an extra primary membrane load being introduced perpendicular to the Bree loadings, Ref. [6]. The analyses in Refs. [2–6] used the same approach as Bree's original analysis, Ref. [1]. However, alternative, "non-cycling", methods for analytical ratchet boundary determination are also emerging, e.g., Refs. [7,8].

The difficulty of obtaining analytic solutions for more complicated geometries or loadings has prompted the development of numerical techniques to address ratcheting and shakedown. For example, direct cyclic analysis methods, e.g., Ref. [9], can calculate the stabilised steady-state response of structures with far less computational effort than full step-by-step analysis. A technique which is now being used widely is the Linear Matching Method (LMM), e.g., Ref. [10]. LMM is distinguished from other simplified methods in ensuring that both equilibrium and compatibility are satisfied at each stage.

This paper presents a Bree-type analysis of the ratchet and shakedown boundaries for the case of a thin cylinder composed of elastic-perfectly plastic material with internal pressure and capped ends, plus an additional axial load (*F*), together with a global bending load. The pressure and additional axial loads are constant primary loads. The global bending is secondary in nature and cycles. The global bending load is envisaged as arising from a uniform diametral temperature gradient with bending of the pipe being restrained. The temperature gradient cycles between zero and some maximum value. Loadings of this type are common in engineering practice. For example, AGR reactor penetrations can develop such diametral temperature gradients. Moreover, thermally driven, cyclic global bending stresses occur generically in steam pipework of any kind.

After developing the analytic solution for the ratchet and shakedown boundaries, the solution is verified by use of the LMM technique. (Alternatively this may be seen as a validation of the LMM technique).

Section 2 formulates the equations which specify the problem. Section 3 defines normalised, dimensionless quantities which will be used throughout the rest of the paper. Section 4 describes the method of solution. Sections 5 and 6 present the solution for the case of tensile ratcheting in the axial direction. Section 7 completes the solution, considering shakedown and stable plastic cycling as well as compressive ratcheting in the axial direction. Section 8 describes the numerical analyses carried out using the LMM method, and finally the key results are summarised in the Conclusions, Section 9.

2. Formulation of the problem

The notation for stresses and strains in this section will include a tilde, e.g., $\tilde{\sigma}$, to distinguish them from the normalised, dimensionless quantities which will be used hereafter. Geometrical linearity, i.e., small strain theory, is assumed.

The problem considers a thin cylinder so that through-wall stress variations may be neglected. The cylinder is under internal pressure, *P*, and an axial load, *F*. Note that capped ends ensure that the pressure load also contributes to the total axial load. Both these primary loads are constant (i.e., not cycling). The cylinder wall is therefore subject to a constant hoop stress, which is uniform around the circumference, of,

$$\tilde{\sigma}_H = \frac{\mathrm{Pr}}{t} \tag{1}$$

where *r*, *t* are the cylinder radius and thickness respectively. The integral of the axial stress around the cylinder circumference equilibrates the applied axial load plus the axial pressure load,

$$F_{TOT} = F + \pi r^2 P = \int \tilde{\sigma} \cdot r t d\theta \tag{2}$$

where $\tilde{\sigma}$ is the axial stress at the angular position θ around the circumference, and the integral is carried out over the whole circumference. Equ. (2) holds at all times since the pressure and the additional axial load are constant.

The axial stress is not uniform around the circumference as a consequence of the cycling secondary bending load. This bending load is envisaged as arising due to a uniform diametral temperature gradient, i.e., a temperature which varies linearly with the Cartesian coordinate $\tilde{x}$ perpendicular to the cylinder axis. Bending of the cylinder is taken to be restrained so that the temperature gradient generates a secondary bending stress. The origin of $\tilde{x}$ is taken to be the cylinder axis. The elastically calculated bending stress is denoted $\tilde{\sigma}_b$, and its tensile side is taken to be $\tilde{x} > 0$. Hence the elastic bending stress distribution across the pipe diameter would be $\tilde{\sigma}_b \tilde{x}/r$ where $-r \le \tilde{x} \le r$. This secondary bending load cycles between zero and its maximum value and back again repeatedly.

The material is taken to be elastic-perfectly plastic with yield strength σ_y. This is a common simplifying assumption in such ratcheting analyses, without which the problem would not be analytically tractable. The Tresca yield criterion is assumed, again for analytic simplicity. Throughout it will be assumed that the compressive radial stress on the inner surface is negligible compared with the other stresses, i.e., the thin shell limit. Stressing is therefore biaxial.

The hoop stress is necessarily less than yield, $\tilde{\sigma}_H < \sigma_y$ otherwise the cylinder collapses. It is worth spelling out why this remains true for this situation of biaxial stressing. Possible cases are.

- If the axial stress, $\tilde{\sigma}$, is positive but less than $\tilde{\sigma}_H$ then the Tresca yield criterion is just $\tilde{\sigma}_H = \sigma_y$;
- If the axial stress, $\tilde{\sigma}$, is positive and greater than $\tilde{\sigma}_H$ then the Tresca yield criterion is just $\tilde{\sigma} = \sigma_y$ and hence yielding occurs with $\tilde{\sigma}_H < \sigma_y$;
- If the axial stress, $\tilde{\sigma}$, is negative then the Tresca yield criterion is $\tilde{\sigma}_H - \tilde{\sigma} = \sigma_y$ and hence yielding again occurs with $\tilde{\sigma}_H < \sigma_y$.

In all cases, therefore, avoidance of collapse requires $\tilde{\sigma}_H < \sigma_y$ as a necessary but not sufficient condition. Consequently, if $\tilde{\sigma}_H < \sigma_y$ is assumed, in regions where the axial stress, $\tilde{\sigma}$, is positive, the yield criterion can be taken to be simply $\tilde{\sigma} = \sigma_y$. In regions where the axial stress, $\tilde{\sigma}$, is negative, the yield criterion becomes $\tilde{\sigma} = -\sigma_y + \tilde{\sigma}_H$. Defining the positive quantity $\sigma'_y = \sigma_y - \tilde{\sigma}_H$, the yield criteria are,

$$\text{For } \tilde{\sigma} > 0: \quad \tilde{\sigma} = \sigma_y \tag{3a}$$

$$\text{For } \tilde{\sigma} < 0: \quad \tilde{\sigma} = -\sigma'_y \tag{3b}$$

The dimensionless parameter α is defined as,

$$\alpha = \frac{\sigma'_y}{\sigma_y} = 1 - \frac{\tilde{\sigma}_H}{\sigma_y} \tag{4}$$

Thus, α quantifies the influence of the hoop stress on the ratcheting behaviour.

Consistent with common practice for similar ratcheting analyses the results will be expressed in terms of the following dimensionless load parameters,

$$X = \frac{F_{TOT}}{2\pi r t \sigma_y} \quad Y = \frac{\tilde{\sigma}_b}{\sigma_y} \tag{5}$$

Because bending of the cylinder is restrained by assumption, the