International Journal of Pressure Vessels and Piping 110 (2013) 32–41

Contents lists available at SciVerse ScienceDirect

International Journal of Pressure Vessels and Piping

journal homepage: www.elsevier.com/locate/ijpvp

Simplified estimates of the creep crack growth parameter $C(t)$ under primary/secondary stresses using the enhanced reference stress method

Terutaka Fujioka [a,*], Robert A. Ainsworth [b]

[a] Central Research Institute of Electric Power Industry, Yokosuka 240-0196, Japan
[b] The University of Manchester, Manchester M13 9PL, UK

Keywords:
Fracture mechanics
Creep crack growth
Non-steady state creep
Simplified estimate

A B S T R A C T

This paper describes simplified methods to estimate the fracture mechanics parameter, $C(t)$, related to creep crack growth rates in non-steady state creep conditions produced by primary or secondary stresses. The methods proposed incorporate effects from initial plasticity and redistribution during a short period after a loading dwell starts, in addition to the estimate of the steady state creep crack growth parameter C^* by the enhanced reference stress method. The methods have been validated by performing finite element elastic–plastic creep analyses of a circumferentially cracked cylinder subjected to load-controlled tension or thermal loads.

1. Introduction

Assessment of crack-like defects in high temperature power plant components is frequently required to secure safe and reliable operation. To predict crack growth behaviour in the creep regime, a J-integral type fracture mechanics parameter, $C(t)$, needs to be calculated. Estimating $C(t)$ in general conditions, however, needs costly detailed finite element analysis modelling of the cracked geometry, and thus using some simplified methods is favoured in practice.

The steady state crack growth parameter, C^* may be estimated by reference stress methods [1,2]. The effects of initial plasticity on $C(t)$ may be incorporated into the evaluation based on some suggestions for limited conditions, where the stress exponents for plasticity and creep are the same and the loading is a combination of mechanical and thermal stresses [3]. Methods to estimate $C(t)$ for more general conditions including pure thermal stresses are not fully established, although some proposals have been made [4,5].

The methods described in this paper employ the enhanced reference stress method [6] to improve the accuracy of estimates of the steady state (or in secondary stresses quasi-steady state) parameter C^*, and the small-scale creep solution in Ref. [7] to incorporate effects from initial plasticity into $C(t)$. Validation by performing elastic–plastic creep finite element analyses (FEA) of a circumferentially cracked cylinder subjected to load-controlled tension or thermal loads is also reported.

2. Procedures to estimate $C(t)$

2.1. Description of stress–strain relations

Expressing stress–strain relations in power law type equations is convenient as the HRR singularity [8,9] is assumed for the stress field in the crack-tip vicinity. The elastic–plastic constitutive law is expressed by the following Ramberg–Osgood law:

$$\varepsilon = \frac{\sigma}{E} + \varepsilon_0^{p}\left(\frac{\sigma}{\sigma_Y}\right)^m \qquad (1)$$

where σ is stress, and ε is strain in general. Material constants are E (elastic modulus), σ_Y (yield strength), ε_0^p (plastic strain at $\sigma = \sigma_Y$), and m (stress exponent). An elastic-perfectly plastic body may be regarded as a special case with $m \to \infty$.

The creep strain rate is expressed by the following Norton law:

$$\dot{\varepsilon}^c = \varepsilon_0^c\left(\frac{\sigma}{\sigma_Y}\right)^n \qquad (2)$$

where $\dot{\varepsilon}^c$ is creep strain rate, and ε_0^c and n (stress exponent) are material constants.

Hoff's analogy [10] requires that the stress distribution for fully plastic response is identical to that in steady state creep if $m = n$.

* Corresponding author.
E-mail address: fujioka@criepi.denken.or.jp (T. Fujioka).

0308-0161/$ – see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.ijpvp.2013.04.019

Nomenclature

C^*	steady state creep crack growth parameter
$C(t)$	non-steady state creep crack growth parameter
C_o	non-steady state creep crack growth parameter at loading dwell start ($t = 0$)
C_L	net section stress correction factor to compensate for inaccuracy of the stress categorization method
E, E'	elastic modulus, modified elastic modulus
I_n	dimensionless function related to stress and strain distributions in the HRR singular field
J	elastic–plastic J-integral
J^e	elastic J-integral
J_o	initial elastic–plastic J-integral at the start of dwell
J_{FEA}	elastic–plastic J-integral estimated by detailed elastic–plastic FEA
J_{ref}	elastic–plastic J-integral estimated by the original reference stress method
K_I	stress intensity factor
R_{in}	inner radius of cylinder
T_{max}	maximum temperature producing thermal stresses
Z	elastic follow-up factor
a	crack depth
m	stress exponent for plasticity
n	stress exponent for creep
w	wall thickness of cylinder
r	radial distance (in a polar coordinate system)
r_s	radial distance between the skeletal point and the crack-tip
t	loading dwell time
γ	limit load correction factor
ε, ε_{ij}	total strain, total strain tensor
ε^c, ε^c_{ij}	creep strain, creep strain tensor
$\dot{\varepsilon}^c$, $\dot{\varepsilon}^c_{ij}$	creep strain rate, creep strain rate tensor
ε^c_o	creep strain rate at $\sigma = \sigma_Y$
$\dot{\varepsilon}^c_{ref}$	reference creep strain rate at $\sigma = \sigma_{ref}$
ε^p, ε^p_{ij}	plastic strain, plastic strain tensor
ε^p_o	plastic strain at $\sigma = \sigma_Y$
$\tilde{\varepsilon}_{ij}(\theta, n)$	dimensionless function related to the strain distribution in the HRR singular field
θ	angle (in a polar coordinate system)
ν	Poisson's ratio
σ, σ_{ij}	stress, stress tensor
$\overline{\sigma}_b$, $\overline{\sigma}_m$	von Mises type equivalent bending stress, and membrane stress
σ_{ref}	reference stress
$\dot{\sigma}_{ref}$	reference stress rate (differential value of σ_{ref} with respect to time)
$\tilde{\sigma}_{ij}(\theta, n)$	dimensionless function related to the stress distribution in the HRR singular field

When a more complex constitutive law including primary creep is employed and a Norton type law may be used to describe the minimum creep rate in steady state creep, the steady state C^* may be estimated by the assumption of Eq. (2).

2.2. Enhanced reference stress method to estimate C^*

The enhanced reference stress method approximates C^* by the following equations.

$$C^* = \frac{\dot{\varepsilon}^c_{ref}}{\sigma_{ref}/E} J^e \tag{3}$$

where $\dot{\varepsilon}^c_{ref}$ is the reference creep strain rate obtained by substituting the reference stress, σ_{ref} for σ in Eq. (2). The elastic J-integral, J^e, is estimated from the stress intensity factor, K_I, for an elastic body with the same loading conditions,

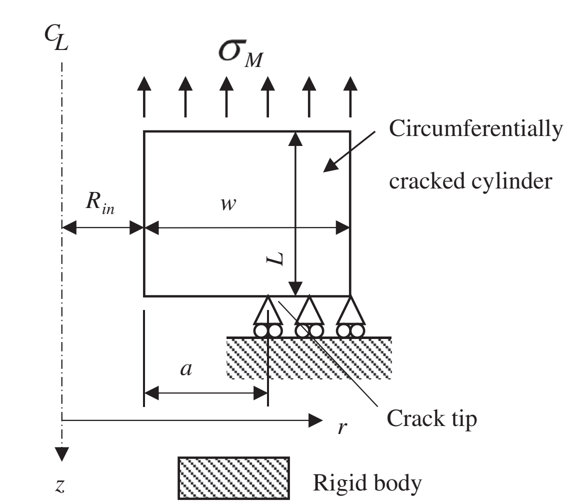

Fig. 1. Circumferentially cracked cylinder subjected to uniform tensile stress.

$$J^e = \frac{K_I^2}{E'} \tag{4}$$

where E' equals E for plane stress, and $E/(1-\nu^2)$ for plane strain conditions (ν: Poisson's ratio).

The reference stress is defined as,

$$\sigma_{ref} = \frac{P}{\gamma P_L}\sigma_Y \tag{5}$$

where P_L is the limit load for the component for a rigid plastic material (equivalent to the assumption of $m \to \infty$ in Eq. (1)), and P is the actual applied load. The limit load correction factor, γ, is non-dimensional, and was introduced by Kim et al. [6] to compensate for inaccuracy of the original reference stress method [11], which may be regarded as a simple case with $\gamma = 1$. The limit load may be accurately estimated by performing FEA of an elastic-perfectly plastic body with increasing P until the cracked component collapses.

The reference stress method using limit analysis is embodied in codes such as R5 [12], although R5 also contains simplified expressions to estimate the reference stress based on the stress categorization concept:

Table 1
Cases analysed, dimensions of the cylinder, and applied primary stress.

Case no.	Case 1 MN	Case 2 MN	Case 1 MB	Case 2 MB
Wall thickness, w	10 mm			
Inner radius, R_{in}	100 mm			
Half cylinder length, L	100 mm			
Dimensionless crack depth, a/w	1/4	1/2	1/4	1/2
Tensile stress, σ_M	98 MPa			
Creep law	Secondary only		Primary plus secondary	