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Simplified estimate of elastic–plastic J-integral of cracked components subjected to secondary stresses by the enhanced reference stress method and elastic follow-up factors

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A B S T R A C T

This paper describes simplified methods to estimate the elastic–plastic J-integral, J, related to the crack growth rate in elastic–plastic situations. Estimating this parameter under general conditions entails costly detailed elastic–plastic FEA modelling of the cracked component concerned, and thus, some simplified methods that do not involve complex numerical calculations are required, particularly, for use in situations where plastic strains are produced by secondary stresses. For mechanical primary stresses, the reference stress method may provide reasonable estimates of J. The direct use of the reference stress method for secondary stresses, however, has not yet been fully established. The method presented in this paper is based on the enhanced reference stress method, which leads to more accurate estimates of J than the original method, and elastic follow-up factors for approximating the inelastic response of the component from the elastic FEA. The present method has been validated by performing detailed elastic–plastic FEA of cracked plates subjected to displacement-controlled loading and of a circumferentially cracked cylinder subjected to thermal loads.

1. Introduction

Assessment of crack-like defects in high temperature power components is frequently required to ensure safe and reliable operation. To predict crack growth behaviours in elastic–plastic situations, a J-integral type fracture mechanics parameter, J is needed to estimate. Estimating J in general situations, however, needs costly detailed elastic–plastic finite element analysis (FEA) modelling of the cracked geometry concerned. Performing these detailed analyses, however, is not practical in many cases; in particular, in the case of fatigue crack growth under cyclic loading entailing the crack closure. Therefore, some simplified methods are required for practical applications.

Representative simplified methods for estimating these inelastic fracture mechanics parameters were reviewed in Ref. [1], and two representative methods, the reference stress approach [2] and the fully plastic solution approach [3], were proposed for crack growth evaluations under creep-fatigue loading. These methods, however, were primarily developed for mechanical loading, and it is difficult to apply these to thermally loaded components. This point is of significance, because secondary stresses exceed the elastic region in some cases of high-temperature plant situations, whereas primary stresses are kept low in accordance with the design codes [4].

A suggestion for estimating relaxation behaviour of a creep crack growth parameter C^* under constant displacement was presented in Ref. [1], where an elastic follow-up factor may be used for simply estimating the relaxation behaviour of the reference stress from the elastic analysis. However, the paper [1] did not refer to any concrete procedure for determining appropriate elastic follow-up factors. A proposal of using elastic follow-up factors to estimate the reference stress due to secondary stresses was later made in Ref. [5]. The definition for the reference stress given in Ref. [5], however, was theoretically different from that of the original reference stress method, and the formulae presented to estimate the elastic follow-up factors were developed for a non-cracked cantilever. This method could not, therefore, be deemed universally valid.

The author presented [6,7] an appropriate method to determine the elastic follow-up factor based on the strictly original definition of the reference stress for displacement-controlled loading, and showed that the estimated factors follow a saturating trend as the applied displacement is increased. The author then theoretically proved that the elastic follow-up factor for inelastic response in a power law-type elastic–plastic body of general shape subjected to a single displacement-controlled load shows the saturating trend,

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Nomenclature

C_L	net section stress correction factor to compensate for inaccuracy of the simplified limit load estimates
E, E'	elastic modulus, and modified elastic modulus, respectively
F	dimensionless stress intensity factor
J	elastic–plastic J-integral
J^e	elastic J-integral
J_{FEA}	elastic–plastic J-integral estimated by detailed elastic–plastic FEA
J_{ref}	elastic–plastic J-integral estimated by the original reference stress method
K_I	stress intensity factor
L	plate length
P	applied load or reaction force
P^e	applied load or reaction force for an elastic body
P_L	limit load
R_{in}	inner radius of a cylinder
T_{max}	maximum temperature producing thermal stresses
Z	elastic follow-up factor
a	crack depth
m	stress exponent for plasticity
n	stress exponent for creep
w	wall thickness of a cylinder
γ	limit load correction factor
ε	total strain
ε^c	creep strain
$\dot{\varepsilon}^c$	creep strain rate
ε_o^c	creep strain rate at $\sigma = \sigma_Y$
$\dot{\varepsilon}_{ref}^c$	reference creep strain rate at $\sigma = \sigma_{ref}$
ε^p	plastic strain
ε_o^p	plastic strain at $\sigma = \sigma_Y$
ν	Poisson's ratio
σ	stress
$\dot{\sigma}$	rate of changing stress
$\overline{\sigma}_b$, $\overline{\sigma}_m$	von Mises type equivalent bending stress and membrane stress, respectively
σ_n	nominal stress
σ_{ref}	reference stress
σ_{ref}^e	elastically estimated reference stress
$\dot{\sigma}_{ref}$	reference stress rate (differential value of σ_{ref} with respect to time)
σ_Y	yield strength

and that the factor converges to a unique value depending on the body shape, loading pattern, and stress exponent [8]. The accuracy of the original reference stress approach, however, deteriorated in the numerical validations presented in Refs. [6,7], and the validations seemed insufficient.

This paper improves upon the author's previous methods in Refs. [6,7] by employing the enhanced reference stress method proposed in Korea [9]. The definition of the reference stresses for thermal loads given in R5 [10] is employed for thermal loads. The methods presented in this paper have been validated by performing detailed elastic–plastic FEA of cracked plates in displacement-controlled loading and of a cracked cylinder subjected to thermal loads.

2. Procedures to estimate *J*

2.1. Description of stress–strain relations

The elastic–plastic constitutive law is expressed by the following Ramberg–Osgood law [11]:

$$\varepsilon = \frac{\sigma}{E} + \varepsilon_o^p \left(\frac{\sigma}{\sigma_Y}\right)^m, \tag{1}$$

where σ is stress and ε is strain, and E (elastic modulus), σ_Y (yield strength), ε_o^p (plastic strain at $\sigma = \sigma_Y$), and m (stress exponent) are material constants. An elastic-perfectly plastic body may be regarded as a special case with $m \to \infty$.

The creep strain rate is expressed by the following Norton law:

$$\dot{\varepsilon}^c = \varepsilon_o^c \left(\frac{\sigma}{\sigma_Y}\right)^n, \tag{2}$$

where $\dot{\varepsilon}^c$ is creep strain rate and ε_o^c creep strain rate at $\sigma = \sigma_Y$, and n (stress exponent) is material constant. Hoff's analogy [12], which defines a similarity between creep and plasticity, requires that the stress distribution for fully plastic response is identical to that in steady state if $m = n$. This characteristic is essential for understanding the later-described definition of elastic follow-up factor for reference stress and strain.

2.2. Enhanced reference stress method to estimate *J*

Both the original and the enhanced reference stress methods approximate J using the following equation:

$$J = \frac{\varepsilon_{ref}}{\sigma_{ref}/E} J^e, \tag{3}$$

where ε_{ref} is the reference creep strain rate given by substituting σ with the reference stress, σ_{ref}, in eqn. (1). The elastic J-integral, J^e, is estimated from the stress intensity factor, K_I, for in an elastic body under the same loading conditions:

$$J^e = \frac{K_I^2}{E'}, \tag{4}$$

where E' equals E for plane stress and $E/(1-\nu^2)$ for plane strain conditions (ν: Poisson's ratio).

The reference stress in the enhanced method is defined as:

$$\sigma_{ref} = \frac{P}{\gamma P_L} \sigma_Y, \tag{5}$$

where P_L is the limit load for the component concerned of a rigid plastic material, and P is the actually applied load. The limit load correction factor, γ, that is non-dimensional, was introduced by Kim et al. [9] to compensate for the inaccuracy of the original reference stress method. The original reference stress method may be regarded as a simple case with $\gamma = 1$. The value of P_L may be accurately estimated by performing FEA of an elastic-perfectly plastic body, where $m \to \infty$ in eqn. (1), with increasing applied load until the component shows collapsing behaviour.

R5 [10] contains simplified expressions to estimate the reference stress based on the stress categorization concept [13]. A revised version of this method may be expressed as follows:

$$\sigma_{ref} = \left\{ \frac{\overline{\sigma}_b}{3} + \sqrt{\overline{\sigma}_m + \left(\frac{\overline{\sigma}_b}{3}\right)^2} \right\} \Big/ \gamma C_L, \tag{6}$$

where $\overline{\sigma}_b$ is the equivalent bending stress and $\overline{\sigma}_m$, the equivalent membrane stress on the cracked ligament. R5 employs an