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Oblique effects on *J* integral of cracked plate under plastic fatigue loading based on inelastic Fe analysis

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A B S T R A C T

This paper describes the features of *J* integrals for cracked plates under plastic loading. The inelastic FE analysis incorporating the accurate constitutive equation provides the detailed inelastic deformations, and leading to show the *J* integral for cracked plates. The FE analysis can give the detailed inelastic deformations so as to produce the accurate *J* integral.

In the present paper, the plate having the straight crack located in the center of the plate, and the plate with one circular hole in the center having the crack at the hole side extending vertically with a slightly oblique direction. The loadings are assumed to be both of monotonic and cyclic force-controlled loading and cyclic displacement-controlled loading. The strain and stress contours are obtained, and effects of different types of force and displacement-controlled loading are investigated in relation to *J* integral development. The effects of oblique cracks, which are observed in the fatigue test at the elevated temperature, will also be studied from the detailed FE analysis.

1. Introduction

Since the *J* integral, introduced by Rice [1], expresses the energy release rate, this fracture parameter is used for the crack propagation rate in low cycle fatigue problem, as is discussed by Dowling and Begley [2], and Dowling [3]. The crack propagation rate is measured and summarized by using *J* integral. Namely, the crack propagation rate da/dN, where a is the crack length and N is the cycle number, can be summarized by the increment of *J* integral, estimated from crack opening to the maximum load based on the experiments. This increment of *J* integral is easily nad accurately estimated by the reference stress method, as is discussed by Webster and Ainsworth [4].

Since the inelastic deformation can be calculated by the FE analysis based on the constitutive equation taking account of cyclic plasticity, the *J* integral also can be calculated by the inelastic analysis. In the present paper, the *J* integral is calculated for cyclic fatigue problem. As the constitutive model, the present paper employs the internal time theory of plasticity by the Watanabe and Atluri [5,6], and this model could explain structural inelastic deformation, as is seen in Ref. [7], and developed to creep problem in Ref. [8].

On the other hand, the present authors [9] carried out fatigue test of the perforated plate at the elevated temperature. The crack propagation rate is summarized by the increment *J* integral estimated by the simplified method by Dowling [2] using stress intensity range and the experimental hysteresis loop. The stress intensity range is estimated by the nominal stress away from the crack tip, even if the test is displacement-controlled loading. The crack propagation rate is estimated by the photographs of the test specimens (Fig. 1) showing the crack shape. However, the crack near the hole side is generally not extended to horizontally, and directed a slightly oblique direction. The crack length is projected at the horizontal axis and measured by the present authors in Ref. [9]. These procedures are not confirmed yet. This paper will investigate the oblique crack effects on *J* integral from the inelastic FE analysis. And the procedure to estimate *J* integral by the nominal stress is also investigated.

2. Experimental results

The fatigue test in Ref. [9] is carried out by use of a Shimadzu servo-hydraulic test machine and a control device. An electric furnace is incorporated in the testing machine for elevated temperature, and it is controlled by the programmed history, and elevated temperature is taken to be constant at 550 °C.

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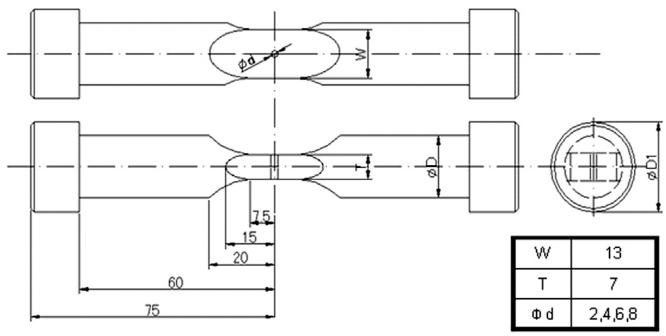
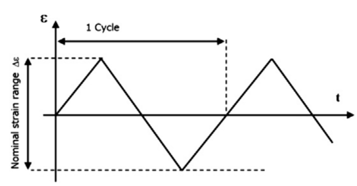


Fig. 1. Test specimen used in the experiment.

Table 1
Summary of experimental condition.

Temperature	550 °C
Strain amplitude	0.3% or 0.5%
Strain rate	0.1%/s
Gauge length	12.5 mm
Fatigue test	

The CCD video camera having zoom lens is set on electric furnace for monitoring crack initiation and propagation. Laborious procedures associated with optical detection of crack have been reduced by using the automated system of CCD video camera. The present CCD video camera records photographs of crack appeared at the hole side of specimens. The strain rate applied for specimens is taken as 0.1%/s in all the specimens. The strain amplitude is 0.5% or 0.3%, and the applied strain shape takes a form of triangle. The test conditions were summarized in Table 1.

Fig. 2 shows pictures of crack propagation history for C1–2 under 0.5% and 0.3% straining. The dominant crack appears at the early stage of 0.5 Nf in C1–2. This is due to that the large stress concentration and small radius of circular hole hastens dominant crack appearance. The orientation of dominant crack is slightly inclined with horizontal line.

When the dominant crack length is measured, the crack path is projected on the horizontal line as shown in Fig. 3. The length in the horizontal direction is defined as the crack length.

Fig. 4 shows the relation of crack propagation rate and increment of J integral by considering closure effects. Every data exists within the factor of 3. From Fig. 4, the following relation is obtained by using the unit of $\mathrm{d}a/\mathrm{d}N$ is mm/cycle, and ΔJ_f is MPa mm

$$\left(\frac{\mathrm{d}a}{\mathrm{d}N}\right)_\mathrm{f} = 2.30\times10^{-5}\left(\Delta J_\mathrm{f}\right)^{1.73} \tag{1}$$

3. Constitutive equation of plasticity

The notation of the stress $\boldsymbol{\sigma}$, the deviatoric stress $\mathbf{S}$ calculated by $\mathbf{S}=\boldsymbol{\sigma}-\mathbf{I}(\mathrm{tr}\,\boldsymbol{\sigma})/3$, the total strain $\boldsymbol{\varepsilon}$, the deviatoric strain $\mathbf{e}$ calculated by $\mathbf{e}=\boldsymbol{\varepsilon}-\mathbf{I}(\mathrm{tr}\,\boldsymbol{\varepsilon})/3$ is used, respectively. The deviatoric strain is denoted by η. The differential strain tensor $\mathrm{d}\boldsymbol{\varepsilon}$ is written as $\mathrm{d}\boldsymbol{\varepsilon}=(\mathrm{d}\mathbf{e}^\mathrm{e}+\mathrm{d}\boldsymbol{\eta})+\mathrm{d}\varepsilon^\mathrm{e}_\mathrm{m}\mathbf{I}$, and the inelastic strain is distorsional. Inelastic behavior of metal alloy can be discussed in terms of deviatoric component, and further $\mathrm{d}\boldsymbol{\eta}=\mathrm{d}\mathbf{e}-\mathrm{d}\mathbf{S}/2\mu_0$, where μ_0 is elastic shear modulus.

The constitutive equation used in the present analysis is the internal time theory of plasticity studied by Watanabe and Atluri

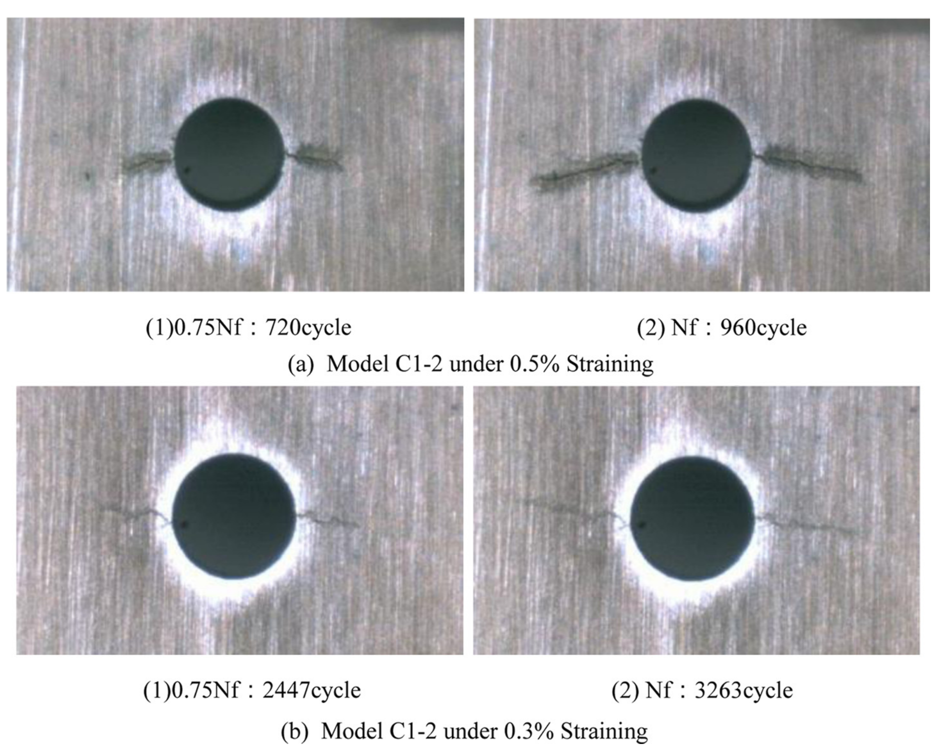


Fig. 2. Experimental observation of crack propagation for Model C1–2.