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Optical properties of an elliptic quantum ring: Eccentricity and electric field effects

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ABSTRACT

We have theoretically studied the electronic and optical properties of a GaAs/AlGaAs elliptic quantum ring under in-plane electric field. The effects of an eccentric internal barrier -placed along the electric field direction, chosen as *x*-axis- and incident light polarization are particularly taken into account. The one-electron energy spectrum and wave functions are found using the adiabatic approximation and the finite element method within the effective-mass model. We show that it is possible to repair the structural distortion by applying an appropriate in-plane electric field, and the compensation is almost complete for all electronic states under study. For both concentric and eccentric quantum ring the intraband optical properties are very sensitive to the electric field and probe laser polarization. As expected, in the systems with eccentricity distortions the energy spectrum, as well as the optical response, strongly depends on the direction of the externally applied electric field, an effect that can be used as a signature of ring eccentricity. We demonstrated the possibility of generating second harmonic response at double resonance condition for incident light polarized along the *x*-axis if the electric field or/and eccentric barrier break the inversion symmetry. Also, strong third harmonic signal can be generated at triple resonance condition for a specific interval of electric field values when using *y*-polarized light.

1. Introduction

Among nanoscale semiconductor heterostructures of varying shapes and sizes, quantum dots (QDs) and quantum rings (QRs) have been attractive low-dimensional semiconductor structures for both theoretical study and potential applications in optoelectronic devices. Previous theoretical and experimental works have shown that, due to their unique topology, QRs behave differently from QDs, with distinct but equally rich electronic and optical properties. Currently, there is a considerable interest in the investigation of QRs due to some intriguing quantum effects, such as the Aharonov–Bohm (AB) oscillations [1], large excitonic permanent dipole moments [2], persistent currents under magnetic response [3] and their practical applications in quantum information processing [4] and terahertz devices [5].

Most of the theoretical studies about QRs [6–10] and analytical models that describe experimental observed effects assume a circular shaped structure. However, although almost perfect circular or slightly oval-shaped QRs have been fabricated [11,12], anisotropic samples of notable elongated QRs have also been synthesized [13,14]. In this connection, the theoretical works on semiconductor QRs have demonstrated interesting phenomena coming from non-circular geometries obtained by eccentricity of the inner hole [15], the presence of transverse barriers [16], the ellipticity [17,18] or more general ring deformations [19,20], as well as by doping with off-centre donor impurities [21–24].

Beyond such passive tailoring of properties during growth, an active control of nanostructure properties through external fields can be easily achieved. Here we shall mention, for example, investigations on the magnetic field effect on the electronic states and second order nonlinear optical rectification coefficient associated with impurity transitions in a QR [23,25]. The energy levels and linear and nonlinear optical absorption coefficients in a two-dimensional GaAs QR subjected to hydrostatic pressure are analyzed by Restrepo et al. [26]. An analytical approach to the problem of an off-centre impurity in a circular-shaped semiconductor QR under crossed axially directed magnetic and radially

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directed electric fields has been reported by Monozon and Schmelcher [27]. Second and third harmonic generation was studied in GaAs based QR under magnetic field in the presence of on-centre impurity [28] or in 3D coupled quantum dot-rings under applied electric field by Duque and his coworkers [29]. Effects of external magnetic fields and the temperature on the second and third harmonic generations in a quantum ring in the presence of simultaneous Rashba and Dresselhaus spin-orbit interactions are presented by Zamani et al. [30].

Interesting studies on anisotropic rings in the presence of external applied fields are also reported. In this context, we note the study of elliptical QRs and of perfectly circular structures including one or more barriers subjected to axial or in-plane magnetic fields [31]. To more clearly analyze the geometrical effects on AB oscillations, elliptical QRs composed of concentric ellipses and the systems where the internal barrier has cylindrical symmetry and the external one is a concentric ellipse are investigated [32]. For both structures it is observed that the AB oscillations decrease in amplitude when the ellipse is elongated. Llorens et al. [33] performed numerical calculations of electronic states in QRs in the presence of an in-plane electric field. The interplay between in-plane applied electric and eccentricity in semiconductor QRs was analyzed by Lavenère-Wanderley et al. [34]. Also, the effects of the eccentricity and in-plane electric fields on the electronic spectrum of a GaAs quantum ring in a perpendicular magnetic field are studied in Ref. [15]. It was shown that the electric field and the eccentricity may suppress the AB oscillations of the lower energy levels.

Eccentric QRs appear also as nanostructures suitable to exhibit interesting optical phenomena and, particularly, they can be considered as natural candidates to achieve large non-linear susceptibilities. These materials have an intrinsic asymmetrical shape in the potential profile that provides the required symmetry breaking to observe second-order non-linear susceptibility. Moreover, the asymmetry can be enhanced either by applying an electric field or under off-centre impurities influence. This behaviour yields the possibility to obtain significant enhancements of the corresponding optical responses compared with other low-dimensional systems and bulk materials.

Deviations from perfect symmetry in these complex structures make necessary the use of numerical approaches to obtain a good description of the energy spectrum, and relative little work has been published on the optical properties in such systems. Grochol and Zimmermann [35] studied transition energies and oscillator strengths of excitons in non-circular semiconductor nanorings of type I and II under applied magnetic field. The adiabatic limit within the infinite barrier approximation is used to study the effect of the external magnetic field on the spectral properties of one-electron in non-uniform quantum ring with radially directed hills [36]. Optical absorption and electro-absorption related to electronic and single dopant transitions in holey elliptical GaAs quantum dots have reported by Vinasco et al. [37]. Very recently, we presented a study related to the impurity and eccentricity effects on the nonlinear optical rectification in a circular-shaped QR under lateral electric fields [38]. We found a striking property of the system: its anisotropic optical response to the linearly polarized light that makes QR a natural candidate for optical devices that manipulate the light polarization.

The present paper extends our previous work on the nonlinear optical absorption, second and third harmonic generation in a circular-shaped two-dimensional quantum ring under axially applied magnetic field [39]. Attention is focused on the effects of the eccentricity and in-plane electric field on the optical nonlinear responses of a particular case of QR, modelled by elliptical QDs with internal axially symmetric cavity.

The paper is organized as follows: In Sec. 2, we describe our model of a QR, presenting the general statements used to solve the Schrödinger equation for electron motion. The results for the electronic states and optical properties (nonlinear optical absorption, second and third harmonic generation) associated to the transitions between energy levels in concentric and eccentric QRs are discussed in Section 3. Finally, Section 4 summarizes the main conclusions of our investigation.

2. Theory

We consider a flat elliptic QR of height h, inner radius R_1 and R_x, R_y the ellipse semiaxes in the xy plane, submitted to a lateral applied electric field F.

The confining potential in the xy plane combines inverse square potential function acting at low distances from the ring centre with a parabolic function at larger distances:

$$V(\vec{r}) = \frac{\hbar^2}{2m^*}\frac{\lambda^2}{(x-d)^2+y^2} + \frac{1}{2}m^*\left(\omega_{0x}^2x^2+\omega_{0y}^2y^2\right). \quad (1)$$

Here, $\vec{r} = (x, y)$ is the in-plane vector position for the electron, the distance d is the eccentricity of the ring. When $d = 0$ the ring is said to be concentric, otherwise it is eccentric. The dimensionless parameter λ characterizes the strength of the inverse square potential which describes the forbidden hollow region inside the ring [7,40–42] and ω_{0x}, ω_{0y} represents the confinement frequencies of the parabolic potential. The inner radius $R_1 = \sqrt{\hbar^2\lambda^2/2m^*V_0}$ corresponds to the value of r where, in a concentric ring, the first term is equal to V_0 and the outer ellipse semiaxes $R_{x(y)} = \sqrt{2V_0/m^*\omega_{0x(y)}^2}$ are associated with the second term of the potential [42]. Here V_0 is the barrier potential for electrons in GaAs surrounded by a $Ga_{1-x}Al_xAs$ material.

If the condition $R_{x(y)} - R_1 >> h$ is fulfilled, we can use the adiabatic approximation to decouple the motion along the z-axis from the xy one. Therefore, the main features of the electron spectrum are essentially determined by the electron motion in the xy-plane [42–44]. In the framework of the effective mass approximation and choosing the electric field direction as x-axis ($\vec{F} = F\hat{x}$), the single-particle Hamiltonian is

$$H = H_0 + eFx \quad (2)$$

where

$$H_0 = -\frac{\hbar^2}{2m^*}\Delta + V\left(\vec{r}\right) \quad (3)$$

Here m^* the electron effective mass, Δ is the two-dimensional Laplace operator and e is the absolute value of the electron charge.

Even for the case of concentric rings, the application of an in-plane electric field breaks the axial symmetry, so that the Hamiltonian (2) does not allow analytical solutions for the eigenfunctions. In order to obtain the eigenenergies E_j and the corresponding wavefunctions $\Psi_j(x,y)$ of the perturbed Hamiltonian we performed here numerical calculations using a finite element method [45] to solve the Schrödinger equation under the effective mass approximation. For eigenvalues and eigenvectors computations we used the geometry of a circle with radius 100 nm with Dirichlet boundary condition. The convergence criterion was set to a relative tolerance of 10^{-6} and the results were get after 3 mesh refinements. The electronic and optical properties were obtained after post-